

David's Midterm 1 Discussion Question Solutions , Cosmology, Fall 2002

1. Mini-summary of the observational evidence and how they relate to the simple Big Bang model:

The Big Bang (BB) model predicts the whole universe was once in a very hot initial state. Today, we observe this as the Cosmic Microwave Background (CMB). The CMB is formed by photons which have decoupled from matter. This event is called recombination, and it occurred roughly 300,000 years after the BB. The expansion of the universe has greatly *red-shifted* the originally very energetic photons. Now, they have very low energies and are in the *microwave*.

The CMB is incredibly uniform (smooth) in temperature. In fact, it is too uniform to explain how the current large scale structures (superclusters and clusters of galaxies) could have developed so quickly. Density fluctuations are required in the early universe before recombination to account for structure formation today. If the early universe contained normal matter, such fluctuations would have caused temperature differences in the CMB which are more than 1000 times greater than what we actually observe. Thus, this suggests some form of matter we can not see, or *dark matter*, which does not interact very much with light, must have existed in the early universe. The dark matter can begin to clump before recombination without causing large temperature fluctuations, because dark matter does not readily interact with light. Thus, the CMB can be very uniform, but at the same time, significant clumping of (dark) matter can occur, enough to "seed" the current large-scale structure in the universe.

Theoretically (using the Adiabatic Theory), we predict the CMB has patches having slightly different temperatures. The angular sizes of these patches is calculated to be about 1 angular degree. When we observe the CMB (for example, with instruments such as BOOMERANG), we see these fluctuations to have sizes to *be* 1 angular degree. Thus, we know we live in a universe which has a flat geometry, and thus Ω_t is 1.

By studying the motions (dynamics) of galaxies in clusters of galaxies, we estimate the total amount of mass necessary to keep the cluster stable is about 10^{15} solar masses. On the other hand, when we count up the galaxies visually (thus accounting for the *visible* matter present in the cluster), we can find only about 3×10^{13} solar masses. This suggests a large amount of matter exists in the clusters which we *cannot* see. We call this *dark matter*.

From our visual studies of cluster of galaxies, we can estimate that Ω_b

(baryonic, the matter we *can* see) is 0.05. However, from our dynamical studies of cluster of galaxies, we estimate that Ω_m must be about 0.3 (the matter that must be present to keep the cluster gravitationally bound). Thus, the dark matter contribution to Ω_m must be about 0.25, since $\Omega_m = \Omega_b + \Omega_{nb}$.

The SNe Ia data suggests the universe is actually *accelerating*, not slowing down. This suggests a repulsive force acts to increase the rate at which the universe is expanding. Since we do not know what is causing such an acceleration, we call it *dark energy*. Dark energy helps to account for the difference in Ω , since we think we live in a flat universe from the CMB data ($\Omega_t = 1$), and yet, our best estimates of how much matter exists only gives us $\Omega_m = 0.3$. So, $\Omega_\Lambda = 0.7$.

That is,

$$\Omega_t = \Omega_m + \Omega_\Lambda \tag{1}$$

$$= \Omega_b + \Omega_{nb} + \Omega_\Lambda \tag{2}$$

$$= 0.05 + 0.25 + 0.7 \tag{3}$$

$$= 1 \tag{4}$$

2.

(a) A star twice as far away will appear 4 times fainter, since the flux falls off as the inverse of the distance squared. That is,

$$\frac{F_A}{F_B} = \frac{D_B^2}{D_A^2} \quad (5)$$

$$= \frac{D_B^2}{(2D_B)^2} \quad (6)$$

$$= \frac{D_B^2}{4D_B^2} \quad (7)$$

$$= \frac{1}{4} \quad (8)$$

(b) If normal dust were in the line of sight, the light would be scattered by the dust and thus appear *reddish*. For the same reason, sunsets look red. (This has nothing to do with *red-shift*. This is simply *reddening* of the light.)

(c) Taken at face value, the SNe Ia results suggest that the universe is accelerating, because we see objects to be fainter than we expect. If the universe really is accelerating, we have to deal with this unknown *dark energy*. However, if the extra faintness can be explained by simply assuming intervening dust scattered and thus attenuated the light, we have a "way out" of having to accept the existence of dark energy. The problem is, if the light really were scattered by normal dust, the light would be reddened due to the scattering. We don't see such reddening in the SNe Ia data. Thus, this dust must be a special kind of dust that doesn't cause reddening when it scatters light. Call this kind of dust *gray dust*. The problem is, no evidence of dust with this special quality exists!

3. (Graphs are in your book.)

A universe with a smaller value of Ω_m has less matter in it to slow the expansion of the universe. Therefore, for a given time, such a universe will have a *larger* scale factor R than a universe with a larger value of Ω_m . Because R is greater, an object at a given redshift z will appear fainter in the universe with a smaller value of Ω_m .

Moreover, an accelerating universe (Ω_Λ is positive) will have an *even* greater scale factor, since its expansion is actually speeding up. Greater R means the distance is greater, and an object at a given z will appear even fainter.

4. Gravitational lensing is the bending (focusing) of light due to the presence of matter in the line of sight between the source of the light (a background galaxy or quasar) and the observer (on earth). Light has energy, and thus by Einstein's $E=mc^2$, has a mass equivalence. Thus, even light is gravitationally attracted to massive objects. Light travels along the shortest path in the fabric of space-time. Mass bends the fabric of space-time. Thus, the path of light is altered when it passes massive objects.

That is:

Matter (energy) tells space (time) how to curve; and
Space (time) tells matter (energy) how to move.

Gravitational lensing is used to estimate the mass of the object causing the lensing, such as a galaxy or a cluster of galaxies. By carefully reconstructing the distorted images of background objects, the mass of the object causing the gravitational lensing can be estimated.

Applied to the universe as a whole, the entire mass-energy of the universe acts to fold space-time to produce its geometry. If the total density is greater than the critical value ($\Omega > 1$), then the universe is closed ($k = +1$) and has a spherical geometry. If the total density is equal to the critical value ($\Omega = 1$), then the universe is flat ($k = 0$) and has a planar (Euclidean) geometry. Finally, if the total density is less than the critical value ($\Omega < 1$), then the universe is open ($k = -1$) and has a saddle-like geometry. In this sense, the whole universe acts like a gravitational lens!