David's Midterm 2 Discussion Question Solutions , Cosmology, Fall 2002

1. Discuss the physical difference between the meaning of Ω_m (matter) and q. How are they related to each other (mathematically) in the simple case of $\Omega_\Lambda = 0$? What additional equations and assumptions do we need to add to the escape equation to derive this relationship between q and Ω_m?

Answer:

(4 points:)

Ω_m is the ratio of the actual density of the universe and the critical density necessary to stop the current rate of expansion. It is a measure of how much matter is in the universe.

q is the de-acceleration parameter, a unitless quantity representing $\ddot{R}$, that is, how fast the universe is slowing down or speeding up its expansion. It is a measure of the dynamics of the universe.

While very different physically, Ω_m and q tell you the same information when Ω_Λ is zero. The more matter in the universe (i.e. the greater is Ω_m), the faster it will slow the expansion of the universe (i.e. the greater is q).

(1 point):

When $\Omega_\Lambda = 0$, $q_0 = \frac{1}{2}\Omega_{m0}$.

(4 points):

We need the following two things to derive the relationship between q and Ω_m with the escape equation:

i.) $dU = dQ - pdV$ (the First Law of Thermodynamics)

ii.) $dQ = 0$ (the Adiabatic Condition).

2. Draw curves of R(t) versus t (starting from t = 0) for a few cases: (a) $\dot{R} = 0$ (static), (b) R = ct, (c) a closed universe for the case with $\Lambda = 0$, (d) a Lemaitre universe, (e) a de Sitter universe.

Answer:

On the R vs. t plot ...

a. $\dot{R} = 0$ is a flat, horizontal line. (1 point)

b. R = ct is a line with a slope of c. (Figure 10.9 (b), p.283 in your text) (2 points)

c. A closed universe w/ $\Lambda = 0$. (Figure 11.8, p. 311, dashed line) (2 points)

d. A Lemaitre universe. (Figure 11.8, p. 311, solid line) (2 points)

e. A de Sitter universe (Figure 11.5, p. 308, q = -1, dotted line) (2 points)

3. Discuss what Big Bang nucleosynthesis models tell us about $\Omega_{m,b}$. How does that compare to current observations which tell us Ω_{m0} is about 0.3? What does this imply about the very existence of non-baryonic matter and its value of Ω ($\Omega_{m,nb}$)? (Include measurements of the observed $\frac{He}{H}$ or $\frac{D}{H}$ ratio in your discussion.)

Answer:

(3 points:)

Big Bang nucleosynthesis models tell us that in the early universe, $\Omega_{m,b}$ was about 0.1.

(3 points:)

We know $\Omega_{m,b}$ was about 0.1 in the early universe from our current measurements of the cosmic abundances of hydrogen (H), helium (He), and deuterium (D). For instance, $\frac{He}{H}$ is about $\frac{1}{3}$ (that is, 25% He and 75% H). Also, $\frac{D}{H}$ is about 1 to 4 x $10^{-}4$ (or 2 D atoms per 100,000 H atom). The deuterium ratio is particularly important, because D is very sensitive to temperature and normal (baryonic) matter density. Too much density or too high temperatures destroys D. The D we observe today must have been left over from Big Bang nucleosynthesis, because the D formed today in stars is rapidly destroyed or used to make other elements. Thus, the abundance of D can be used as an independent method to estimate $\Omega_{m,nb}$ shortly after the Big Bang.

(3 points:)

Studies of the dynamics of galaxy clusters today suggest Ω_m is about 0.3. However, Big Bang nucleosynthesis tells us the normal baryonic contribution to Ω_m is only 0.1. That is, Big Bang nucleosynthesis tells us too little normal baryonic matter formed initially to be able to account for the mass we infer must exist from studying galaxy motions. Thus, the balance must be *non-baryonic* matter, which does not interact strongly with normal matter. This "dark matter" could be present in greater quantities in the early universe without destroying D, because it does not interact with D very much.

Since $\Omega_m = \Omega_{m,b} + \Omega_{m,nb}$, and we know that $\Omega_m = 0.3$ (from dynamical studies of galaxy clusters) and $\Omega_{m,b} = 0.1$ (from Big Bang nucleosynthesis), then $\Omega_{m,nb}$ must be about 0.2.

4. Discuss how inflation relates to the ordinary Big Bang in terms of the evolution of the scale factor of the Universe. Discuss how inflation is used to explain the relative uniformity of the CMB and how this relates to the concept of "casual contact."

Answer:

(3 points:)

Inflation exponentially expands the scale factor R by a factor of 10^{50} in 10^{-32} seconds. Two points virtually touching each other are dragged apart much faster than the speed of light during the inflation event, powered by a positive cosmological constant. The ordinary Big Bang has no such event.

(6 points:)

Inflation is necessary to explain how two points on opposite sides of the sky can have essentially the same temperature today. That is, it resolves the "horizon problem." With the ordinary Big Bang, the two points are already 10 million light years apart by the time of recombination (when the brick wall became transparent and the liberated photons formed the CMB). Thus, the two points were not in "causal contact." That is, even at the speed of light, information (about each other's temperatures) could not be communicated between them fast enough to account for why they are at the same temperature today. And this is true for *every* opposite pairs of points in the sky! It would be like if you invited 20,000 people to a party and they all wore the exact same red tee-shirts. You would know some kind of communication must have occurred beforehand!

Inflation resolves this problem. Before the inflation event, the universe was only 10^{-25} m in size. So, any two points *were* in causal contact with each other. The exponential expansion of the inflation event then dragged every point away from each other much faster than the speed of light to distances out of causal contact with each other. But they still have the same temperature because 10^{-32}s earlier, they *were* in causal contact with each other. Thus, as these two points are beginning to enter our visual horizon today, we can understand why they have same temperature without having to simply assume that uniform temperature was an arbitrary initial condition for the universe at the time of recombination.

(See Appendix A, Homework Solution Set 6 for more discussion.)