<u>David's Final Exam Discussion Question Solutions , Cosmology, Fall 2002</u>

1. Based on the uniform disk model and the Keplarian model, graph how the rotational velocity of an object in a galaxy should depend on its distance to the center of the galaxy. Then, graph what is actually observed in real galaxies. Discuss these graphs in terms of the "cosmic conspiracy" and dark matter.

(4 points): The graph: On the y axis is the rotational velocity (v). On the x axis is the distance (R) from the center of the galaxy. The uniform disk model predicts a linear increase in rotational velocity the farther away from the galaxy's center, up to the visible edge of the galaxy. After that, the Keplarian model predicts a $\frac{1}{R}$ decrease. What is actually observed in real galaxies: no Keplarian decrease, but a constant leveling off of rotational velocities beyond the visible edge, even far out beyond the galaxy's visible edge.

(4 points): The discussion: Based on the simple picture of gravity, we expect that the rotational velocity of an object in a galaxy should depend on its position in relation to the center of the galaxy. Up to the visible edge of the galaxy, it should orbit around the center of the galaxy faster the farther away it is from the center. This is because for distances farther away from the center, more and more mass (assuming a uniform disk) is inside the object's orbit. More mass means greater gravitational force, and hence, a greater velocity.

Beyond the galaxy's visible edge, no additional matter can be seen. Thus, the total mass of the galaxy has apparently been reached, fixing the gravitational force to be constant for distances beyond. Since gravity falls off as $\frac{1}{R^2}$, the farther away an object is from the galaxy's center, the less gravity it experiences, and hence, the slower its orbital velocity. Thus, by the Keplarian model, an object's rotational velocity should diminish beyond the visible edge of the galaxy.

However, in reality, 21 cm line observations of gas clouds beyond the visible edge of a galaxy shows no decrease in the rotational velocity beyond a galaxy's visible edge! The velocity levels off and becomes constant beyond the edge. Something is keeping the object orbiting faster than is expected. The simplest explanation is that this "something" is *matter*, since as far as we know, only matter causes gravity, and only gravity keeps objects orbiting around another mass. Since we can't see this matter, it is not luminous and *dark*. So, it is *dark matter*. Furthermore, its arrangement in space must keep objects orbiting at a constant velocity the farther away it is from the galaxy's center. This unexpected and odd behavior is the "cosmic conspiracy."

2. This figure shows the age of the universe in units of $1/H_o$ versus Ω_m. Discuss the meaning of the two curves. Include in your discussion why the dotted curve helps resolve the apparent paradox that, before we knew about dark energy, some objects in the universe were apparently older than the universe itself.

(3 points):

The solid curve assumes no cosmological constant ($\Omega_\Lambda = 0$). For a given value of the Hubble constant H_o, the age of the universe increases for decreasing Ω_m.

The dotted curve includes a cosmological constant, but assumes a flat universe, so that

$$\Omega_t = \Omega_m + \Omega_\Lambda = 1. \tag{1}$$

(2 points):

As Ω_m decreases, Ω_Λ increases to keep the sum equal to 1. Thus, for smaller values of Ω_m, the universe can experience more acceleration. In effect, this increases the possible age of the universe, because for the same value of the Hubble constant, the universe could have began earlier. (Recall the R vs t plot for a universe with Λ ... it starts to shoot up again after leveling off, so we can match the same slope after the point when the universe accelerates.) Thus, a universe with a cosmological constant can be older than a universe without a cosmological constant but still have the same Ω_m. As the plot shows, for the same value of Ω_m, the dotted line (which includes Λ) corresponds to a larger value of $\frac{t}{H_o^{-1}}$ than the solid line (which has no Λ).

(3 points):

Including the cosmological constant helps resolve an old paradox regarding ages. Before we knew the universe might be accelerating, some objects in the universe (globular clusters) were estimated to be older than the universe itself. This would be as silly as if a child were older than the parent! But the SN Ia results suggested that perhaps the universe was accelerating, that is, it might have a non-zero cosmological constant. This allowed us to revise our estimate of the age of the universe to a larger value, larger than the age of the old globular clusters. Thus, including the cosmological constant resolved the age paradox.

3. Discuss QSOs. Include in your discussion the concepts of flux, luminosity, redshift, time variability, causal contact, and the source of their power.

(2 points):

QSO stands for *quasi-stellar object.* A QSO looks like a star in photographs, but it has a very large redshift, implying (via the Hubble Relation) it lies at a very great distance d from us. That we can even see it implies it must be extremely bright. Indeed, when we measure its flux (apparent brightness) F, and use the flux relationship ($F = \frac{L}{4\pi d^2}$), we find that a typical QSO's luminosity L is on the order of a trillion of our suns (about 10-1000 average galaxies)! In itself, this is incredible, but then consider its inferred size and the picture becomes even more bizarre.

(3 points):

The light from QSOs fluctuates over a period of about one week. The concept of causal contact says that since nothing can travel faster than light, the fastest way one part of an object can communicate information to another part is at the speed of light. In terms of fluctuations, causal contact helps us estimate an object's size, because no object can coherently grow brighter and dimmer in less time than it takes light to travel across it. Thus, a QSO's fluctuation period of a week implies its maximum size is a light-week in size. Compared to the size of an average galaxy (about 100,000 light-*years*), this is *extremely* small.

(3 points):

So, here is an object with the luminosity of about 1000 average galaxies but only a tiny fraction of the size of an average galaxy! Normal nuclear fusion (which powers stars like the sun) simply cannot account for such a huge amount of energy. Rather, it takes a supermassive black hole to explain the tremendous luminosity of QSOs. A QSO is believed to be an example of an early galaxy with a supermassive (millions of solar masses) black hole at its core. A black hole is an object which is so dense that even light cannot escape from its gravitational pull. The strong gravity of the black hole accretes gas and dust onto it, forming an accretion disk around the black hole. The material in the disk is swirling so fast due to the tremendous gravity that it becomes very hot and therefore emits a lot of energy. Furthermore, as the dust and gas approach the black hole, some of it gets ejected away (like from a sling-shot) from the black hole, forming long jets. (The interaction of ions with magnetic fields in the jets create powerful synchrotron radiation, making them very bright.)

4. Describe how you would demonstrate to a friend using a piece of paper and pencil how the method of parallax works. (That is, reproduce the demo we did in class.) How does parallax explain why humans are unable to accurately gauge distances (with the naked eye) beyond a distance of about 50 feet? Why is parallax important in cosmology?

(3 points):

i) Draw a dark line down the center of the paper.

ii) Hold the paper at arms length.

iii) Hold the pencil some distance closer to your face than the paper.

iv) Alternatively close your right or left eye, keeping the other open. Notice how the position of the pencil as seen relative to the dark line on the paper seems to shift as you alternatively close each eye. This is the parallax effect.

v) Hold the pencil right up to the paper and repeat step iv). Now, the pencil doesn't seem to move with respect to the dark line, because it is at the same distance as the line of reference.

(2 points):

Human depth perception depends on the left and right eyes being able to perceive an object's slightly different positions with respect to background objects. Of course, this depends on the "baseline" distance between the left and right eyes. Beyond a certain point, the perceived angle to an object is so small that it appears at the same position to both eyes. Beyond this point (about 50 feet), humans no longer gauge distance accurately.

(3 points):

The pencil is like a nearby star, the line on the paper like the position of a background star which is so far away its position appears fixed, and the position of your eyes like the opposite positions of the Earth in its orbit around the sun. Viewing the nearby star at one time in the year, we notice it is positioned a certain way against the background star. Half a year later, the star seems to have moved with respect to the background star. The angular distance it moves is twice the parallax angle p. From simple geometry, the distance to the star is $d = \frac{1}{p}$, where p is in arc seconds and d is in parsecs.

Thus, the parallax method is used to find distances to nearby objects. It forms the 2nd lowest rung of the distance ladder. Parallax (which is itself based on radar ranging to precisely determine the baseline distance of the Earth to the Sun) is used to callibrate the Cepheids, which in turn is used to calibrate higher methods, etc., etc. Adjusting our parallax measurements (as HIPPARCOS did) may significantly affect all our distance measurements, potentially changing our view of the universe, e.g. its age and evolution.