David's Solutions to Homework Set 1, Cosmology, Fall 2002

1. Use the equation $T = T_0(1+z)$

Solving for T_0, we have

$$\begin{aligned} T_0 &= \frac{T}{(1+z)} && (1) \\ &= \frac{3000}{2000} && (2) \\ &= 1.5 && (3) \end{aligned}$$

2. The cosmic microwave background (CMB) is so uniform in temperature that it cannot account for the present large scale structure of the universe. The CMB reveals the state of the early universe at the time of recombination. From the temperature variations in the CMB, we can infer the magnitude of density fluctuations in the early universe. If the only matter in the universe was normal matter, then the density fluctuations inferred from the CMB is far too small to account for the large scale structures in the universe we see today.

There is a relationship between the temperature of the CMB and density of normal matter because photons were coupled to electrons (normal matter) before the event called *recombination.* Before recombination, the universe was so hot that it was filled with free electrons. Photons of all energies can interact with free electrons. Thus, a photon would not get very far before being absorbed and re-emitted by another free electron. It would be like trying to get across a crowded room, but constantly bumping into people. It would take a long time for you to get across the room. Likewise, photons were constantly *scattered* by free electrons in the early universe. Photons were thus *coupled* to normal matter. This is why Prof. Ulmer describes the CMB as the "brick wall": you can't see beyond it! For the same reason, you can't see through a cloud; water molecules scatter photons, preventing them from reaching your eyes in a coherent manner to convey what was behind the cloud.

When the universe finally cooled enough to form neutral hydrogen atoms, electrons were no longer free. Rather, they became bound to protons, thus forming hydrogen atoms. This event is called *recombination.* Another way to describe recombination is to say that photons became *decoupled* from normal matter. In order to interact with hydrogen, a photon must have a very specific energy. So after recombination, photons of all other energies could now stream freely away and no longer be constantly scattered. The were liberated (decoupled), and we can see them today in the form of the CMB.

During recombination, higher density regions would be evidenced by slightly cooler temperatures, whereas lower density regions would be slightly hotter. This is because higher density regions have more mass, and therefore deeper gravitational potential wells. It takes energy for photons to "climb" out of deeper gravitational wells. Thus, photons escaping from higher density regions would lose more energy and reveal themselves as cooler regions in the CMB. Likewise, lower density regions would be hotter spots in the CMB, since they didn't have to lose so much energy. Since the difference in temperature between hot and cold regions in the CMB are very small, the differences in density must also have been very small.

Fluctuations in density are required for some parts of the universe to begin contracting to form large scale structures. These fluctuation in density can thus be considered "seeds" of structure in the universe. In order for massive structures such as superclusters and clusters of galaxies to form, the density fluctuations of normal matter should be 10-100 times larger than what is revealed in the CMB. That is, the temperature variations in the CMB are expected to be 10-100 times greater than what are actually observed. That is, if the only matter in the universe was normal matter (fluctuations of which would have revealed themselves as temperature differences in the CMB), then we can not account for the large scale structures in the universe today.

However, if we postulate the existence of a special kind of matter which interacts significantly less with photons than normal matter, then we have a way out of this dilemma. We can say that this special matter (call it *dark matter* since it doesn't interact with radiation) *can* begin clumping significantly without revealing the clumping in the CMB. The temperature variations would not reveal the true extent of dark matter density fluctuations, since photons do not interact with the dark matter. After recombination, normal matter would be gravitationally attracted to the already significant pockets of dark matter, and eventually form the large scale structures we see today.

Thus by invoking dark matter, we remove the inconsistency between the smoothness of the temperature of the CMB (which suggests very little density differences of normal matter) and the need for significant density fluctuations to account for the current large scale structures in the universe today (since it was dark matter that began clumping significantly before recombination, and this clumping would not be revealed as temperature differences in the CMB).

3. The first peak in the CMB fluctuation, assuming adiabatic fluctuations, tells us the universe is flat.

By assuming adiabatic fluctuations, we are able to directly relate the temperature fluctuations in the CMB to the actual, normal matter density fluctuations. The adiabatic theory says "the light follows the matter", so where there are temperature differences, there are density differences. The adiabatic theory also predicts the largest angular size of the regions which have temperature fluctuations to be 1 degree. And we actually observe them to be 1 degree. So, light did not bend significantly as it traveled through the universe from the "brick wall" (the CMB) to us. Thus, the universe must have a flat geometry.

If the universe had a closed, spherical geometry, the light rays would bend toward each other as they traveled through space. For instance, two people starting out on the equator of the Earth walking parallel due north will actually converge at the North Pole. This is because the Earth is a sphere, and parallel lines at the equator all meet at the pole. Thus, if we lived in a universe with a close, spherical geometry, two light rays would converge. If we expected something at the far edge of the universe to be 1 degree in size, we would actually observe it to be *greater* than 1 degree. Similarly, if the universe had an open, saddle geometry, light rays would bend outward away from each other as they traveled through space, and we would see something to be *less* than 1 degree which we expected to be 1 degree. Only in a flat universe, where the light rays do not bend at all as it travels through the universe, would something expected to be 1 degree actually be observed to be 1 degree. This is what we see, so we live in a flat universe.

4. Hot dark matter (HDM) goes with the top-down scenario for galaxy and cluster of galaxy formation. That is, large superclusters of galaxies formed first, then fragmented into clusters of galaxies, and then galaxies. An example of HDM is the neutrino. It travels very fast (close to light speed), and is thus *hot.* Each neutrino is not very massive, but there may be so many of them that they can account for much mass taken together as a whole.

5. Cold dark matter (CDM) goes with the bottom-up scenario. That is, smaller clouds of stars merged to form galaxies, which then became gravitationally bound to form clusters of galaxies, and then finally superclusters of galaxies. An example of CDM are the weakly interacting massive particles (WIMPs), which barely move, and are thus *cold.* Observations of the early universe indicates smaller objects existed, and so support the CDM scenario.

However, numerical simulations with CDM do not produce enough superclusters and voids which we observe in the real universe today.

6. The total mass density of the universe affects the curvature of the universe because Einstein said: (a) the amount of mass concentration affects space-time curvature both locally and the universe as a whole.

General relativity in a nutshell:

Mass tells Space how to curve, and
Space tells Mass how to move.

7. Discussed in Problem 2. The addition of dark energy is needed because from CMB measurements, we know the universe is flat. Thus, $\Omega = 1$. However, when we measure the amount of normal matter as well as infer the amount of matter which must exist, we get only $\Omega_m = 0.3$ for all kinds of matter, normal and dark. The difference must be made up by something other than mass. But since we don't know what it really is, we call it *dark energy.* So,

$$\begin{aligned} \Omega_t &= \Omega_m + \Omega_\Lambda \qquad (4) \\ &= \Omega_{m,baryonic} + \Omega_{m,non-baryonic} + \Omega_\Lambda \qquad (5) \end{aligned}$$

8. The Microwave Anisotropy Probe (MAP) results will be reported in January 2003. This is an all-sky survey of the CMB (matching the coverage of the Cosmic Background Explorer COBE), but at very great angular resolutions (matching the resolution of Balloon Observations of Millimetric Extragalactic Radiation and Geomagnetics BOOMERANG). It will verify whether the 1 degree peak is truly at 1 degrees for the entire CMB. The current BOOMERANG data which is used to show the 1 degree peak was obtained from studying a very small (35^o x25^o) patch of the sky.

	COBE	BOOMERANG	MAP
Type	Satellite	Balloon	Satellite
Launch Date	1989	1998	2001
Resolution	7^o	0.2^o	0.3^o
Extent of map	Whole sky	35^o x 25^o patch	Whole sky