David's Solutions to Homework Set 2, Cosmology, Fall 2002

1. In terms of the equation of state ($P/\rho$), the general dark energy version is $P/\rho < 0$, while Einstein's original version is $P/\rho = -1$.

2. A *white dwarf star* is the *remnant* of a star which originally had a mass around $10M_\odot$ or less. The remnant of a star is whatever is left over after normal nuclear burning has ceased in a star, that is, after the star has stopped evolving. Another way to think of the remnant is the "corpse" of a star.

| Star's Original Mass | Core Remnant |
|---:|---:|
| Low Mass ($\sim \frac{1}{4}M_\odot$) | Helium White Dwarf |
| Medium Mass ($\sim 1-5M_\odot$) | Carbon White Dwarf |
| High Mass ($\sim 6-8M_\odot$) | Neon-Oxygen White Dwarf |
| Higher Mass ($\sim 10-20M_\odot$) | Neutron Star |
| Very High Mass ($> 25M_\odot$) | Black Hole |

3. To make a Type Ia Supernova(SN Ia), we need a *carbon white dwarf* in a *binary system.*

Because a white dwarf is not burning nuclear fuel, no gas pressure exists to resist the inward pull of gravity. Hence, the white dwarf contracts until the electrons cannot get any closer to each other. Pauli's exclusion principle states that no two electrons can occupy the same state (position in space). For the same reason, you can't walk through walls. This kind of pressure keeping electrons apart is called *electron degeneracy pressure*, and the white dwarf's material is in a *degenerate* condition. (You and everything solid around you are also degenerate in this sense of the word.)

However, if the white dwarf accretes (pulls) material onto itself from a binary companion, then the mass of the white dwarf grows. But the star cannot continue accreting mass indefinitely, because the more mass it acquires, the stronger gravity pulls inward on itself. Electron degeneracy pressure cannot balance gravity if the mass exceeds $1.4M_\odot$. If the mass exceeds this limit (called the *Chandrasekhar Limit*), the white dwarf collapses. The temperature rises extremely rapidly, igniting carbon nearly simultaneously. That is, the white dwarf explodes and we see this event as a Type Ia Supernova (SN Ia).

4. We can calibrate SN Ia because we assume all carbon white dwarfs which become SNe Ia have the same amount of mass when they explode. If

the underlying physical mechanism is what was described in Problem 3, SN Ia are triggered when the mass of a white dwarf reaches the *Chandrasekhar Limit* of $1.4M_\odot$. It is like igniting a one-gallon tank of gas. Because you know the tank has one gallon, you know how bright it ought to burn. Likewise, we assume all SN Ia have the same amount of "fuel", namely, $1.4M_\odot$ worth of material. Thus, we theoretically know the intrinsic luminosity of SNe Ia, and we can use it to find the distance to the object using the flux relationship $F = \frac{L}{4\pi d^2}$. The flux is simply the object's apparent brightness as seen from the Earth.

5. The maximum *mass* a white *dwarf* can have before it explodes is *1.4*$M_\odot$. This is the Chandrasekhar Limit.

6. Given that we measure accurately the age of the universe, how would we expect the expansion rate to depend on the value of the total mass density $\Omega_m$ of the universe?

The expansion rate decreases with increasing $\Omega_m$. This makes intuitive sense, because the more matter in the universe (and thus a higher $\Omega_m$), the faster the universe will slow down its expansion. The greatest possible expansion rate is one which is a fixed constant (never decreasing) and this corresponds to a *null universe* without any matter in it ($\Omega_m = 0$). Introduce more matter (thus increasing $\Omega_m$) and the expansion rate decreases with time. That is, in a real universe with mass and thus a source of gravity, the expansion rate will slow down with time. The more mass in the universe, the faster the expansion will slow down.

The analogy of throwing an object straight up in the air helps. During the upward journey (before it reaches the top of its trajectory), the object travels fastest at the moment it leaves your hand. Every moment afterward, the object slows down a little more until at the very top of its trajectory, it momentarily stops before falling back down. If we then stand on an even more massive planet than Earth while throwing the object up, the gravitational pull would be greater and cause the object to slow down even faster (the deceleration is greater). If we were on a less massive planet, the opposite would happen: the object would slow down slower (the deceleration is less). Thus, the more mass (analogous to higher $\Omega_m$), the less the speed of the object (a lesser expansion rate) at any given time (age of the universe).

7. Given a certain expansion rate, how would the scale factor (R) vary

with $\Omega_m$?

For a given expansion rate, the scale factor (R) decreases with increasing $\Omega_m$. Plotting the scale factor (R) versus time clearly shows this relationship. It is also intuitive, since the more mass in the universe (the bigger $\Omega_m$), the faster the universe wants to slow its expansion, that is, have a smaller scale factor (R) at a given expansion rate.

Notice, too, that given a certain expansion rate, the age of the universe corresponding to that expansion rate decreases as $\Omega_m$ increases. That is, the more mass in the universe (the bigger $\Omega_m$), the sooner the expansion rate will approach a certain value different than the one it started out with. For an extreme example, consider again the null universe with $\Omega_m = 0$. Because there is no mass in such a universe, the expansion rate always stays the same and will *never* approach a value other than the one it started out with.

8. When we study SNe Ia, we measure two things: i) its apparent brightness from Earth (i.e. its flux), which tells us its *distance* via the flux relationship (since we think we know L ... see Problems 3 and 4), and ii) its redshift (z), which tells us its recession *velocity* from us. We expect that since the universe was expanding faster in the past (regardless of the value of $\Omega_m$), the velocity versus distance curve should not have a constant slope, but actually curve upward and have a steeper slope for more distant objects. This is simply saying that the universe was expanding faster in the past. That is, if we live in a universe which has only matter in it and is thus always slowing down its expansion due to the attractive force of gravity, the Hubble Constant should be greater in the past.

However, when we actually plot velocity (from z) versus distance (from F and L) of the SNe Ia, we find that curve is actually sloping *downward* in the past! This indicates that the Hubble Constant was actually smaller in the past, that is, the universe was expanding at a *slower* rate than it is today! In other words, the universe has sped up, or *accelerated* recently! If you see an object (like the Space Shuttle) accelerate, you know it has a source of energy (like rockets) attached to it. Likewise, the fact that the universe seems to be accelerating implies the universe has some sort of energy causing it to accelerate. Since we don't know what is causing the acceleration, we call this mysterious energy *dark energy*.

9. We want to use SNe Ia as standard candles, so we need to be confident that SNe Ia are essentially identical events throughout the universe, regardless

of distance. However, the behavior of the light curves of SNe Ia at greater distance (higher z) appear rather different from those which occur nearby. For example, SNe Ia farther away do not reach the same maximum brightness compared to nearby ones. Also, the time it takes for the light to diminish back to pre-supernova levels is longer for SNe Ia with higher z. Taken at face value, these inconsistencies may shake our confidence that SNe Ia can be used as standard candles.

However, we can remedy this apparent problem by factoring in time dilation as postulated by Einstein in his theory of special relativity. According to someone watching from a (relatively) stationary frame of reference, time seems to pass more slowly in a moving object, that is, the passage of time appears to be *dilated*. We on Earth are in a stationary reference frame, and we are measuring events from objects moving away from us due to the expansion of the universe. By Hubble's Relation, the farther away the object, the faster the object is moving. Thus, those objects which are farther away (larger z) should experience more time dilation than those close by. That is, from our perspective on Earth, things appear to happen over a longer period of time on an object which is moving faster away (and thus farther away) from us than another object which is not moving as fast (and thus closer).

Applied to the SNe Ia data, the apparent inconsistencies can be explained when time dilation is taken into account. The delayed light decay and smaller maximum brightness of more distant SNe Ia are both consistent with the expected effects of time dilation. This strengthens not only our appreciation of special relativity but also our confidence that SNe Ia are indeed governed by the same underlying physics and thus can be reliably used as standard candles.

10.

Supernova Differences:

| | Type Ia | Type II |
|---|---|---|
| Object | Carbon White Dwarf | Very Massive Star |
| In a Binary System? | Necessary | Not necessary |
| Original Mass | 1 to 5 $M_\odot$ | $> 12M_\odot$ |
| Mass of Core | $\sim 1.4M_\odot$ | Varies |
| Composition of Core | Carbon/Oxygen | Iron |
| Cause of Explosion | Thermonuclear (Chand. Limit exceeded, ignition of C due to high temperatures) | Gravitational (Lack of gas pressure to counter-balance gravity, since iron can't burn) |
| Spectral Features | Si II, fast light curve decay | $H_\alpha$, slow light curve decay |

11. This is an example of the *Twin Paradox* in special relativity. That is, consider a pair of twins on Earth. Both, of course, are the same age. Let one board a spaceship which travels out to Vega at high velocities, close to the speed of light. Let her turn around from Vega and return home to Earth. When she arrives back, she will be *younger* than her twin!

Time dilation slowed down the traveling twin's clock in relation to the one on Earth. And that includes *biological clocks*, too! You might object that from the point of view of the traveling twin, the twin on Earth was the one who moved. After all, everything is *relative*. However, there is a difference. The twin on the spaceship had to physically *accelerate*, that is, move from a stationary reference frame into a moving reference frame. This results in a definite physical difference between the two twins. The one who travels outward to Vega will be the one who is younger. Now, if the other twin had hopped aboard another spaceship to catch up to the first twin, when they eventually meet, the second twin will be younger than the first one (because he had to be traveling faster in order to catch up)! Ah, the joys of relativity!

For Dr. Arroway in Contact to have travelled to Vega and back in a reasonable time, she would need to travel at a very high speed, a fair fraction of the speed of light. Thus, she would experience time dilation compared to her boyfriend on Earth. By the time she came back, he would have aged much more than she did, causing obvious complications in their relationship, assuming he were still alive.