David's Solutions to Homework Set 3, Cosmology, Fall 2002

1. Clusters of galaxies contain about 1000 galaxies.

Cluster	Number of Member Galaxies
Local Group	450
Virgo Cluster	2,500
Coma Cluster	10,000

2. Cluster of galaxies are about $10^{15} M_{\odot}$ and about 1 Mpc (3Mly) in diameter .

3. The gas we see by its X-ray emission in the inter-cluster medium (ICM) has a temperature of about (a) 10^8 K.

4. We think non-luminous matter exists in clusters of galaxies because when we calculate how much mass should be in a cluster due to its dynamics (for instance, by measuring velocities of constituent galaxies and then equating the resulting centripetal force to the gravitational force), we come up with a number which is 10-100 times more than the mass we can visually account for in the cluster. This extra mass thus must be non-luminous, or dark matter.

But, this dark matter may simply be normal matter which simply doesn't emit light, like rocks, asteroids, dust, gas, planets, burned-out cores of stars (cool white dwarfs, neutron stars), or black holes. This normal (or *baryonic*) matter is one candidate for the dark matter. However, we can use Big Bang nucleosynthesis calculations to estimate how much baryonic matter was created initially in the Big Bang. These calculations yield a value only $\frac{1}{8}$ the amount of mass needed to account for the mass we infer must exist in clusters of galaxies. Thus, this dark matter is not only non-luminous but also must be fundamentally different from normal matter. That is, it must be *non-baryonic* matter.

5. We observe some galaxies in clusters have different recession velocities because (a) some are moving toward us and some are moving away from us. The cluster as a whole is moving away at some group velocity, but the individual galaxies within the cluster may be moving relative to the group. Those moving toward us will seem to be moving at smaller recessional velocities than the group, and vice versa for those moving away.

6. *Gravity* keeps hot gas in the ICM from evaporating and leaving the cluster. In fact, this is further evidence of dark matter: the amount of mass we calculate should be present to keep the hot gas from evaporating is much greater than the amount we actually observe to be there.

7. *Gas pressure due to high temperatures* keeps the hot gas in the ICM from collapsing to the cluster center. Gas pressure is also what keeps our Sun (and all stars) from collapsing inward on itself.

8. Gravitational lensing may be understood from two different perspectives:

i) $E = mc^2$

Einstein equated energy to mass. So, a photon (a particle of light), which has no mass, does however have a *mass equivalent*, because it does have *energy.* So, just like an object having mass, a photon will experience gravitational attraction toward another massive object, such as a cluster of galaxies. The path of light will bend around the massive object just as if the photons were being gravitationally attracted toward it. In fact, the deflection of the path of light from a background star around the Sun was used as one of the first tests of Einstein's theory.

ii) Space-time Geometry

A succinct summary of Einstein's theory of general relativity:

Mass-Energy tells Space-Time how to curve.
Space-Time tells Mass-Energy how to move.

So, a massive object like a cluster of galaxies curves the fabric of space-time, and light follows this curvature of space, thus bending around the object. Analogously, a lens bends light rays and focuses them. The massive object, then, acts as a huge lens, a *gravitational lens.*

9. Looking back in time, we expect if Ω_m is low, then the number density of galaxies should be *greater* than if Ω_m is high.

If Ω_m is low, then large structures such as galaxies would cease to form after a certain point in time. Density decreases as the universe expands. Recall: $\rho = \frac{M}{V}$, and as the universe expands, mass M stays the same, but the volume V gets bigger. After a certain amount of time in a universe with low Ω_m, the density is too low to form new galaxies. So, given the observed number density of galaxies today, if Ω_m is low, this number should have been reached relatively quickly in the universe (when the density was still sufficient to form galaxies). As a result, we expect to see a *greater* number density of galaxies if Ω_m is low.

On the other hand, if Ω_m is high, galaxy formation can continue for a greater period of time. Even though density is decreasing due to the universal expansion, the density necessary for galaxy formation persists for a longer period of time. Thus, given a number density of galaxies today, that value should have been reached relatively later in the universe. (Otherwise, if galaxy formation happened sooner, there would be more time to form galaxies, resulting in a number exceeding what we actually observe today.) That is, galaxy formation in a universe with large Ω_m is a faster event, but it had to have occurred relatively later to account for the number density we observe today. Thus, when we look back in time, we expect to see a *smaller* number density of galaxies if Ω_m is high.

10. Based on our studies of clusters of galaxies, the approximate value of Ω_m is (c) 0.3.

11.

$$\begin{aligned} \Omega_t &= \Omega_m + \Omega_\Lambda && (1) \\ &= \Omega_b + \Omega_{nb} + \Omega_\Lambda && (2) \\ &= 0.05 + 0.25 + 0.7 && (3) \end{aligned}$$