

David's Solutions to Homework Set 6, Cosmology, Fall 2002

1. The Big Bang Model of the Universe explains or is consistent with:

* The finite age of the universe;

* The observation that the universe is expanding (Hubble's Relation).

* The existence and characteristics of the Cosmic Microwave Background (CMB), which is evidence that the entire universe was once very, very uniformly hot;

* The relative abundance of H (75%) and He (25%) in the universe today, which is evidence that the universe was so hot that nucleosynthesis could occur;

* The observation that the sky is *not* white hot today (Olber's Paradox). The universe is finite in size and age, so only a finite number of stars exist in any given direction, and these stars have a finite age. Furthermore, because the universe is expanding as a whole, the energy of the light from these objects, as well as the CMB, is redshifted to lower energies.

2. The *neutrino* (ν) is the particle left over from the Big Bang and would let us see back to about 1 second after the Big Bang.

The neutrino hardly interacts with normal matter and is thus able to pass through all the stuff of the universe unhindered. In contrast, photons (light particles) are scattered by free electrons and were thus coupled tightly to matter, making the universe opaque before recombination. (For the same reason, you can't look through a cloud, because the photons interact with the water molecules and are scattered.)

If you wanted to build a house that was neutrino-free, such that the walls, ceilings and floors blocked out all neutrinos, they would all have to be made out of lead and 1 light-year thick!

Think of neutrinos as what ghosts must be made of. (They can pass through anything!)

The temperature of the background neutrinos from the Big Bang is theoretically predicted to be around 2K. (This is analogous to the temperature of the background photons in the CMB, which temperature is around 3K.) There is a Nobel Prize waiting for you if you detect these background neutrinos, just as the two discoverers of the CMB (Penzias and Wilson) were rewarded with

Nobel Prizes.

3. Olber's Paradox says that if the universe is infinite and stars live forever, our skies should be white hot, because in every direction we look, we would eventually see a star. The Big Bang resolves this paradox by making the universe finite in size and its constituent stars finite in age. Therefore, we don't necessarily come across a star in every direction we look. Furthermore, because the universe is expanding, the light from far away objects (as well as the CMB) is redshifted into lower energies below the visual spectrum, thus making them invisible to our eyes, though still detectable with the proper instrumentation sensitive to those energies.

4. Wherever we look in the universe today, the relative abundance of hydrogen (H) is about 75% and that of helium (He), about 25%. The remarkably uniform distribution of these elements strongly suggests they were formed in a single event at the same time. Otherwise, if they were made in different places at different times, why would the distribution be so uniform throughout the universe?

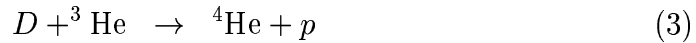
Here is a summary of He nucleosynthesis (p.367-370 in your book):

* 10^{-6} s to 10^{-4} s after the Big Bang, free neutrinos, neutrons, protons, and electrons formed from the hot soup of energy caused by the Big Bang. At this time, protons and neutrons were being interconverted into each other by reactions with neutrinos and anti-neutrinos. As the temperature cooled, though, more protons were produced than neutrons. This is because protons are less massive than neutrons, as well as the fact that less energy is required to form protons from neutrons than the opposite reaction. All told, 1 s after the Big Bang, the percentage of neutrons was about 14% and of protons, 86%.

* The temperature continued to cool and about 180 s after the Big Bang, it was cool enough for neutrons and protons to combine to form an isotope of H called deuterium (D), in the process forming a photon γ . Before this time, the universe was still so hot that highly energetic photons would rip apart D immediately after it formed. After it became cool enough for D to stably form, two D atoms could then combine to form ^3He plus a free neutron. Finally, another D would react with the ^3He to form ^4He , the nucleus of He, plus a free proton.

Schematically, the reactions look like:





* At the same time this was happening, free neutrons were decaying naturally into protons, electrons, and anti-neutrinos. Neutrons are inherently unstable. The half-life of a neutron is only 10.5 minutes, that is, after 10.5 minutes, you are left with only half the number of neutrons you started out with. So, very quickly all the neutrons either reacted with protons in the above manner to form ${}^4\text{He}$, or decayed to protons, electrons and anti-neutrinos. In the end, nearly 25% of all the baryonic matter ended up as ${}^4\text{He}$, and the rest were lone protons, that is the H nucleus. (To be complete, the nuclei of other elements such as lithium (Li) and beryllium (Be) were also formed, but their abundances were very small compared to H and He.) This ratio agrees with the abundances of H and He we see in the universe today, so we believe that most of the H and He were created shortly after the Big Bang in this manner.

* Deuterium (D) is a very sensitive creature. It is easily destroyed when the temperature or density is high. The D which is formed in stars today is quickly destroyed or used to form other elements. Thus, any free D we observe in the universe *must* be primordial, that is, must have been created shortly after the Big Bang. That is, the D we see today must have been the left over by-products of ${}^4\text{He}$ synthesis outlined above. Moreover, since D is very sensitive to the normal baryonic matter density present, measuring D would be an independent gauge of Ω_b (associated with normal, baryonic matter) in the early universe.

* Today, we find roughly 2 atoms of D per 100,000 atoms of H. (Or, as your book puts it, the ratio $\frac{D}{H}$ is about 1 to 4×10^{-5} .) This places a limit on Ω_b to be less than 0.1. That is, if Ω_b is more than 0.1, we should observe less D today, and if it was less, then we should find more D. This measurement provides an independent estimate on Ω_b , and interestingly, it is consistent with the answer from our cluster of galaxy surveys.

5. A *plasma* is a gas made up of free electrons and protons. Such is gas is also called an *ionized* gas. Recall that photons of any energy are scattered by free electrons. So, light cannot readily pass through such a gas. In other words, it is *opaque*. Thus, the CMB appears to be a "brick wall" to us; we can't see through it.

6. Inflation insists that the universe must have started out flat because Ω_m is so close to 1 today. Today, $\Omega_m = 0.3$. If Ω_m were slightly less than 1 at the beginning, it would have rapidly plunged to values much less than 1 as

the universe expanded. If Ω_m were slightly greater than 1 at the beginning, it would grow to values greater than 1 as the universe expanded. Only if Ω_m were exactly equal to 1 would it remain 1. So, if our current Ω_m is so close to 1 today, it must have been *really* close at the beginning. It must have been 1 to an accuracy of 50 decimal places!

Imagine throwing a ball up into the air and after 10 billion years, you still couldn't tell whether the ball would come back down or continue flying away. You must have initially thrown that ball at a speed *awfully* close to its escape velocity! Likewise with our universe. After 10 billion years, Ω_m is still essentially 1, so it must have begun awfully close to 1.

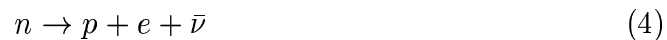
7. A typical bar magnet (and the Earth) has 2 poles.

8. Cut a bar magnet in half and you still have 2 poles. (Another Nobel Prize is waiting for you if you can find a magnet with only 1 pole! See how "easy" it is to get Nobel Prizes!)

How do monopoles relate to cosmology? The hot Big Bang model and GUTs (see Problem 12) predict magnetic monopoles were created in symmetry breaking processes as the universe cooled. For example, the number of magnetic monopoles which should be created during electroweak symmetry breaking (when the electromagnetic force separates from the weak force) is large. So, the Big Bang model predicts magnetic monopoles to be a prevalent part of our universe. And yet, no one has ever discovered a magnetic monopole. Maybe you will be the first!

(Your book calls the absence of magnetic monopoles the *relic problem*. Read about it on pp. 422-423.)

9. One particle the neutron decays into is the *proton*. (The other particles it decays into are the electron and the anti-neutrino.)



10. If neutrons decayed in 1 second rather than 10.5 minutes, He and D would be present in much lower abundances than we find today. (See Problem 4.)

11. The name of the model which Inflation is based on is the *deSitter Model*.

The deSitter Model is a null universe ($\Omega_m = 0$) but with a large cosmological constant Λ , so it accelerates rapidly and grows exponentially. Inflation assumes our universe behaved in a similar manner shortly after the Big Bang.

12. GUTs stand for *Grand Unified Theories*.

There are 4 fundamental forces which have been identified in nature:

- * Gravity (keeps you from falling off the earth)
- * Electromagnetic Force (keeps you from passing through the floor)
- * Weak Force (causes radioactive decays)
- * Strong Force (keeps nuclei together)

GUTs attempt to combine three of these forces into one: the weak, electromagnetic, and strong forces. These forces were one force shortly after the Big Bang, during the GUT era (see Figure 13.8, p. 378 in your text).

A more ambitious endeavor is to combine all the four forces into one force. These are called TOEs (Theories of Everything). See Figure 13.4, p. 366. Trying to include gravity with the other three forces is extremely difficult, because to do so, we need to understand how gravity behaves on the quantum level. Unfortunately, today gravity is so weak at the quantum level that it is virtually impossible to measure its effects. The only time in history when gravity was strong at a quantum level was during the very early stages of the evolution of the universe. Our current particle accelerators simply cannot reproduce such extreme temperatures and energies, however. Thus experimentally testing these regimes is simply out of reach at this time. Nevertheless, some theorists are working on TOEs, and the most promising theories to date are *string theories*. But that's another story for another time.

13. The Harrison Zel'dovich spectrum is "the same-power-on-all-scales perturbation distribution".

(More about this in Problem 15.)

14. Baryogenesis is evoked to explain the imbalance we see today between *photons and baryons*.

(More about this in Problem 16.)

15. (I sensed in class on Monday that there was a lot of confusion regarding the theory of Inflation. You might find help in Appendix A where I present the essence of Inflation.)

In order for us to make any sense of the perturbations in the CMB which we observe today (for instance, the Boomerang data with the peaks at 1 degree and smaller), we have to know the initial conditions of the plasma before recombination occurred. Most simply, we assume the strength ("power") of all perturbations in the plasma were the same for all angular sizes ("scales"). This "same-on-all-scales perturbation distribution" is named after the astrophysicists who first proposed it: Harrison-Zel'dovich. Inflation naturally produces such a distribution ("spectrum"), because it proposes that in a very small time, all size scales in the universe were virtually instantly created, thus giving all of them the same initial condition. Without assuming the Harrison-Zel'dovich spectrum, interpreting the peaks in the CMB data would be impossible, because you wouldn't know the initial conditions of each scale.

An analogy might help. Imagine you have a perfectly smooth piece of paper. The initial condition is simply that the whole piece of paper is perfectly smooth. Now, if you fold the paper and put a crease in it, then you could easily tell where you *perturbed* the paper. You could tell that because you knew the initial conditions: you started with a smooth piece of paper. However, let's say you started out with a piece of paper that already had many creases in it. You put another crease in it. Unless you knew exactly which creases were already there, it would be very difficult to tell which crease you just made. That is, unless you knew the complex initial conditions of the original paper, it would be virtually impossible to make sense of any new perturbations you introduce on the paper.

Likewise with the CMB. By assuming the Harrison-Zel'dovich spectrum (distribution), we assume perturbations on all scales began at the same strength. Then, oscillations in the plasma due to the interaction of baryonic matter, non-baryonic matter, and radiation perturbed the plasma, causing some regions to be more dense than others. When recombination occurs, these over-dense regions would have slightly different temperatures than the under-dense regions. In fact, the over-dense regions would be slightly *cooler*, since the photons lose energy by climbing out of the potential well of the dense regions. And in the "light follows the matter" Adiabatic Theory, these tiny perturbations in temperature would reveal the presence, position, and sizes of the density perturbations. Since we assumed the Harrison-Zel'dovich spectrum, we can now

make sense of these perturbations. We can make meaningful interpretations of the observations to tell us something about the early universe.

More interestingly, only when we assume the Harrison-Zel'dovich spectrum do we get results which resemble the perturbations observed in the CMB today. If we assume any other initial condition, we don't get anything resembling what we actually observe!

Do we *require* Inflation to produce the Harrison-Zel'dovich spectrum? No, but it does give a reasonable physical motivation for why all scales of perturbations would start with the same power. The Harrison-Zel'dovich spectrum was originally proposed long before Inflation, but it was based on statistical and observational evidence. Until Inflation came along, no one had a good physical explanation why the distribution of perturbations would start off with such a "clean slate".

16. Physicists were motivated to invent baryogenesis because they love the idea of *symmetry*. Why would there be an imbalance of matter to antimatter on the order of 1 in a billion? It's not symmetric! Much of physics is incredibly symmetric, and the more romantic among us describe this as *beautiful*. So an asymmetry, no matter how small, is terribly irritating to those who believe nature and its laws are beautiful.

More on baryogenesis in Problem 17.

Discuss why or why not you think this is valid in 100 words or fewer.

It's your turn now ... do you think we should try to understand things based on the (somewhat human) notion that things should be symmetric and even beautiful in the universe?

17.

C = *Charge conjugation*.

The C operator changes a particle's charge. That is, it converts a particle into an anti-particle. Most reactions in nature obey C symmetry, that is, swapping all the matter particles in the reaction with their anti-particle counterparts changes nothing in the reaction. However, some reactions do not obey C symmetry. For instance, applying C to a right-handed neutrino ν_e creates a right-handed anti-neutrino $\bar{\nu}_e$, but no one has ever found a right-handed anti-neutrino; all anti-neutrinos are *left-handed*! So, C by itself is not a perfect

symmetry in nature.

$P = \textit{Parity}$.

The P operator changes the sign of all the coordinates x, y, z. This will change a particle's handedness, for example from left to right. P acts like a mirror, "reflecting" the image of the original particle. P symmetry says that if I change the handedness of a particle, all the physics should remain the same. For example, a bicycle built out of right-handed screws should operate just as normally as one built out of left-handed screws. It would violate P symmetry (and be very strange) if the two bicycles behaved differently! Yet in nature, some P symmetries are violated. Using the neutrino again as an example, we can apply P to to a right-handed neutrino ν_e to form a left-handed neutrino. However, no left-handed neutrinos have ever been found; all neutrinos are *right-handed*! So, P by itself is not a perfect symmetry in nature.

$T = \textit{Time reversal}$

The T operator changes the sign of time in an equation. For instance, it changes positive time (forward) into negative time (backward). Prof. Ulmer gave a good example of the T operator. Apply T to a pendulum and you can't tell the difference. There is no preferred direction of time in the laws of physics. Reversing the direction of time would not violate any fundamental physical laws.

How does CPT relate to baryogenesis?

In the standard Big Bang Model, the energy density was so high in the early universe that matter and anti-matter pairs formed out of the vacuum in a process called pair-production. However, moments after their creation, the matter and anti-matter pairs would annihilate each other and release a high-energy photon. This is the origin of all the photons in the CMB. However, our very existence proves that not *all* the matter was annihilated this way. There must have been a slight excess of normal matter in the early universe. The goal of baryogenesis is to physically explain the origin of this excess.

Here is one (simplified) explanation of baryogenesis by GUTs. After the inflation event, the energy in the universe created X and \bar{X} particles. The X and \bar{X} particles were present in exactly equal numbers (symmetry!). The X particles would transform into quarks (building blocks of normal matter), while the \bar{X} transformed into anti-quarks (building blocks of anti-matter).

$$X \rightleftharpoons q + q \tag{5}$$

$$\bar{X} \rightleftharpoons \bar{q} + \bar{q} \tag{6}$$

However, the transformations of the X particles occurred at a slightly greater rate than the \bar{X} particles. This is an example of CP (charge conjugation and parity) violation. If they were not CP violating, they would occur exactly at the same rate. Of course, we can't simply concoct CP violation to fit our story. CP violation was experimentally observed by James Cronin (U. of Chicago) and Val Fitch (Harvard) in 1964 in a type of particle called the *kaon* K .

$$K \rightarrow \pi^- + e^+ + \nu_e \tag{7}$$

$$K \rightarrow \pi^+ + e^- + \bar{\nu}_e \tag{8}$$

The first reaction occurs 0.2% more frequently than the second. In principle, they should occur at exactly the same rate. The reasoning then follows that if such CP violating reactions exist today, then they must have also existed back in the early history of the universe. Thus, this very real physical mechanism may be responsible for creating the slight excess of matter over anti-matter, and ultimately to our being here thinking about such things!

If CP is violated, then T is also violated. A theorem in GUT says that CPT is never violated. That is, a right-handed matter particle going forward in time is identical to its left-handed anti-matter counterpart going backward in time. But if CP is violated, then T must be also violated to ensure that CPT is not violated. If T is violated, that means that there is an arrow of time at the sub-atomic level! Time's direction may be a fundamental part of our universe, not just a statistical property as the law of entropy suggests. This has profound implications.

In summary, this is the story of baryogenesis:

At first, reactions (5) and (6) occurred equally in both directions, but after the universe cooled enough, there was not enough energy to replace the X s and \bar{X} s. Because reaction (5) occurs slightly faster than reaction (6), slightly more quarks than anti-quarks were created. The universe then cooled further to allow quarks to combine and form baryons, and anti-quarks to form anti-baryons. The slight excess of quarks thus translated into a slight excess of baryons. Then, baryons and anti-baryons collided, annihilating each other in pairs, producing high energy photons which filled the plasma; the CMB is made up of these photons, now greatly redshifted due to the expansion. Virtually all the anti-matter in the universe was thus destroyed. The small, remaining excess of baryons makes up the material universe today.

Some of you may wonder ... so, what have we really gained? We merely replaced one problem by introducing another problem: why is CP violated in

the X and \bar{X} ? By invoking CP, we get rid of having to simply throw up our hands and saying the the universe simply started out with a slight imbalance of normal matter compared to anti-matter for no good reason. Accepting such things is rather unsatisfying. Physicists like to *explain* things, not merely accept things. With the mechanism of CP violation, we can at least start with a perfectly symmetric system and then explain the slight difference in matter vs. anti-matter with a real, experimentally verified, physical phenomenon, We don't have to accept the asymmetry as some magical initial condition without a reason. What causes CP violation? Many are working on that right now. Perhaps you will help figure it out!

18. Big Bang nucleosynthesis calculations of the deuterium abundance limit Ω_b to be about 0.1 in the early universe. From studying the dynamics of galaxies, Ω_m is about 0.3. Thus, there must be Ω_{nb} of about 0.2. This is an independent line of evidence suggesting that non-baryonic dark matter must exist.

Appendix A

The Essence of Inflation

*What is Inflation?

Inflation is a theory which proposes shortly after the Big Bang, our universe rapidly (exponentially) expanded.

*What are the core ideas behind Inflation?

Inflation proposes:

i) 10^{-35} s after the Big Bang, our universe expanded from a tiny 10^{-25} m in diameter to 10^{+24} m, or 30 Mpc, larger than the biggest supercluster we know of today! The whole inflationary event took only 10^{-32} s, but in that time, the universe expanded by an incredible 10^{+50} times! In fact, during inflation, two points in space receded from each other at speeds many orders of magnitudes greater than the speed of light!

ii) The *cosmological constant* Λ dominated the energy density of the universe during the inflation event, which caused its tremendous expansion. (More precisely, this has to do with the freezing out of the strong force as predicted by GUTs, false and true vacuums, and a lot of other techno-babble. Aren't you *so* glad you don't need to worry about them?)

iii) After the inflation event was over, the universe resumed normal expansion, and the energy density decayed to form the matter and radiation that fills the universe today.

*Why do we need a theory like Inflation?

The Big Bang Model explains a lot of things regarding our universe, as suggested in Problem 1. However, it does not explain a number of important questions, including ...

i) Why is the CMB so uniform in temperature? (The horizon problem.)

Without inflation, it's very difficult to understand how two patches of the CMB on the opposite sides of the sky could have the same temperature to an accuracy of 1 in a million parts. These parts of the sky would have been 10 million light years apart by the time of recombination, so how could they "know" they had to have the same temperature? The only way is to physically transfer (or "communicate") the information, and the fastest way to communicate information is at the speed of light. But the two patches are so far apart

that such communication could not take place in time to ensure the CMB was so uniform at the time of recombination. It would be like if you invited 20,000 people to a picnic and they all brought potato salad! That's awfully suspicious and you would know they had to have communicated with each other beforehand to agree to all bring potato salad. Likewise with the CMB. If all patches of the sky are essentially the same temperature, it's reasonable to suspect that they were all once in "communication" with each other. One way to resolve this problem is simply to assume that the universe simply started out at the same temperature everywhere. But that's a big assumption which cosmologists are reluctant to make. Being able to physically explain something is much more satisfying than simply assuming it to be true.

ii) Why is the current value of Ω_m so close to 1? (The flatness problem.)

As we saw in Problem 6, the current value of Ω_m being so close to one required that in the beginning, Ω_m had to be 1 to an extremely high level of accuracy. But why? Was there something about the universe that required Ω_m to be 1 initially?

iii) What caused the initial matter density fluctuations? (The structure problem.)

As we learned before, current large scale structures in the universe (super-clusters and clusters of galaxies) had to have grown from some initial density fluctuations in the early universe. But where did these "seeds" originate?

iv) Why are we (normal matter) here if matter and anti-matter were present in equal quantities at the beginning? (The symmetry problem.)

Matter annihilates with anti-matter, and if they are present in equal quantities at the beginning (as they should be, due to mass-energy conservation), then they should have been completely annihilated; our universe should be nothing but radiation today. Obviously, since we are here, a slight excess of normal matter was present in the early universe. But why?

Inflation offers answers to these questions (and several others).

*How does Inflation solve these problems?

i) Inflation easily solves the horizon problem. Before inflation, two points which would be separated by many millions of light years were actually virtually touching each other, that is, so close that they *could* communicate information between each other, allowing them to establish similar physical properties, such as temperature. Then inflation occurred and separated the

two points at astounding speeds exceeding light speed. Thus after inflation, these two points are much farther apart than they would have been by normal expansion, well out of communication range of each other. But they still share the same properties, because before inflation, they were in contact with each other.

ii) Inflation also readily answers why the universe was essentially flat at the beginning. Imagine blowing up a balloon. At first, when the balloon is small, the curvature of the balloon is obvious. However, after you blow it up *really* big, a local patch on the balloon looks flat. Likewise, the earth under your feet appears to be flat, and it took a long time for humans to realize it is actually a big ball. So, Inflation provides a mechanism by which the early universe was essentially flat. That mechanism is the exponential growth in size which produced an essentially flat geometry. Thus, in a flat universe, Ω is 1, which we know must be true since Ω is so close to 1 today.

iii) The structure problem is also addressed by Inflation. During inflation, the sizes of quantum fluctuations may have been magnified to scales large enough to form the density fluctuations (the seeds) which eventually develop into the large scale structures of the universe. The inflation process naturally produces the Harrison-Zel'dovich spectrum of perturbations, which assumes all scales have the same power. The origin of seeds may also be explained by irregular phase transitions which develop when the forces freeze out, that is, when a symmetry is broken as described by GUTs. Concentrations of matter-energy may form in string-like structures, called *cosmic strings*, and their motion through space-time would cause density perturbations to form.

GUTs actually predict several kinds of discontinuities:

$$\text{Pointlike} = \text{monopoles} \quad (9)$$

$$\text{Linear} = \text{cosmic string} \quad (10)$$

$$\text{Planar} = \text{domain wall} \quad (11)$$

$$\text{Diffuse 3D clump} = \text{texture} \quad (12)$$

None of these have ever been observed. Perhaps you will be the first to do so!

iv) The symmetry problem is resolved by invoking CP violating reactions. See the answer to Problem 17 for more details.