David's Solutions to Homework Set 7, Cosmology, Fall 2002

1. The equation $KE + kc^2/R^2 = PE + \Lambda$ is simply the Friedman (escape) equation with a cosmological constant.

During inflation, the scale factor (R) of the universe explodes from a very, very small value to a very, very large value, due to an extremely large contribution to the energy by Λ.

So, if R becomes very large, the curvature term ($\frac{kc^2}{R^2}$) becomes negligible. Even if k were not actually zero at the beginning, the whole curvature term would become very small and effectively become zero as R exploded. That is, for all practical purposes, k might just as well be zero after inflation. So, after inflation, the geometry of the universe is basically flat.

A flat universe is associated with $\Omega = 1$.

Thus, in the early universe, the geometry is essentially flat (k = 0) and $\Omega = 1$.

As an analogy, think about blowing up a balloon. At first, when the balloon is small, the curvature of the balloon is obvious. However, if you blow up the balloon *really* large (as would happen to the universe during the inflation event), then the curvature is hardly noticeable anymore locally. For the same reason, the earth appears flat to us, even though we now know it is just a big sphere. Likewise, as the universe exploded in size during inflation, the geometry *became* flat, due to the rapid inflation.

2. Simply put, the Brane Theory is an alternative explanation to the origin, evolution, and fate of our universe. So far, we have studied the standard model in cosmology, which involves a hot Big Bang and Inflation. Most cosmologists (99%) accept this model, since it successfully explains most of our observations of the universe. However, the Brane Theory proposes an entirely new explanation. Quite simply, it says we live in an infinitely large, infinitely old, geometrically flat universe, which undergoes an endless cycle of evolution and renewal. It is able to explain all our current observations of the universe (homogeneity, flatness, isotropy, seeds of large scale structure), as well as provide a couple of answers to questions which the standard Big Bang/Inflation Models can not.

The Brane Theory says it is better than Inflation because ...

i) It can explain the role of dark energy today.

In the standard Big Bang/Inflation Models, the recently detected acceleration of the universe (caused by some unknown dark energy) has no natural explanation and must be added ad hoc. The Brane Theory includes dark energy as a fundamental ingredient in the universe. In fact, dark energy is what drives the whole cycle of expansion and collapse in this theory.

ii) It can predict the future of the universe.

In the standard Big Bang/Inflation Models, the future is uncertain. Dark energy has no fundamental role in these theories, and so its long term behavior can not be predicted. If it is a constant (as in Einstein's idea of a cosmological constant), then the universe will continue to expand forever. But if it is something else (such as *quintessence*, a concept that encompasses many different possibilities for the dark energy), then it might decay and end the acceleration. In the Brane Theory, the future is certain: the universe will appear to re-collapse into a "Big Crunch", and then regenerate itself in another "Big Bang", and the cycle will continue forever.

In contrast to Inflation, the Brane Theory predicts:

i) No non-baryonic matter exists.

The Brane Theory proposes that the source of the extra gravitational forces which we measure (for example, from cluster of galaxy dynamics), and which we have been attributing to the unseen, "dark matter", is actually the gravitational interaction between our universe (on one "brane") and a parallel universe (on another "brane"). Thus, there is no such thing as "dark matter", and so, we won't find it.

ii) No background gravitational radiation exists.

In Inflation Theory, the early universe accelerated extremely *rapidly* during the inflation event, and *gravitational forces* played a significant role in this process. Thus the Inflation Theory predicts a background of gravitational radiation should be left over from inflation, analogous to the cosmic background radiation we observe in the Cosmic Microwave Background. We should be able to detect these gravitational waves using instruments being built today (LIGO, LISA, etc.). However, the Brane Theory proposes that the early universe accelerated extremely *slowly*, and hence, gravity plays an insignificant role during that time. Thus, there will be no gravitational wave background. If gravitational waves *are* detected, the Brane Theory will suffer a serious blow.

The philosophy behind the Brane Theory is the *cyclic* or *endless* universe.

(See Appendix B for a simple summary and tabular comparison of the Big Bang/Inflation and Brane Theories.)

3. Putting a spacecraft at the distance of Pluto will certainly increase the baseline of parallax measurements from 1 AU (for Earth) to 40 AU (for Pluto). However, the orbital period of such a satellite will increase from 1 year (for Earth) to 250 years (for Pluto)! Since the satellite would have to travel at least $\frac{1}{4}$ of its orbit in order to obtain the parallax angle, it would take about 60 years to gather the data!

(One rather straightforward way to get around this problem is to send out *two* satellites at the same time, each positioned on opposite sides of the orbit, or $\frac{1}{4}$ of the orbit, if you would like. It would take about 10 years to get the satellites into their positions, after which they can take simultaneous measurements from their individual vantage points. That way, we wouldn't need to wait so long for the data!)

However, there are also *cost* and *technological* factors to consider. While prohibitive, it is in principle not impossible.

Is it worth it? I would think so, seeing how much HIPPARCOS improved our distance measurements!

4. As measured from Earth (baseline = 1 AU), the distance/parallax angle relationship is simply:

$$d = \frac{1}{p} \tag{1}$$

... where

d = the distance to the object in parsecs (pc). (1 pc = 3.26 ly)

p = parallax angle measured in arc seconds.

*** IMPORTANT NOTE *** : the above simple relationship is true only if the parallax angle p is in the units of *arc seconds*. If you're given an angle in terms of a different unit (such as degrees or radians), you must either convert the angle into arc seconds, or use the appropriate equation. See Appendix A.

So, in this problem, p = 0.05 = $\frac{5}{100}$ arc seconds,

$$
\begin{aligned}
d &= \frac{1}{\left(\frac{5}{100}\right)} \qquad (2)\\
&= \frac{100}{5} \qquad (3)\\
&= 20pc \qquad (4)
\end{aligned}
$$

If the baseline were extended ten times (to 10 AU), and we observed the same parallax angle, then the distance simply increases by a factor of ten to 200 pc. You can reason this out by simply drawing a right triangle. To keep the same interior angle, lengthening one leg of the triangle by a certain factor requires that the other leg must also increase by the same factor.

*** ANOTHER IMPORTANT NOTE *** The *larger* the parallax angle, the *closer* the object! And vice versa, of course!

5. You can never *prove* anything in science, but you can certainly *disprove* something. If Inflation predicts WIMPS, and we truly don't find any, then it certainly doesn't bode well for the theory. Of course, proponents of Inflation may simply object and find fault with your experimental/observational method. (For instance, they can claim your technique was not sensitive enough, etc., which may or may not be true, depending on how well you did your work.) At the same time, not finding WIMPS doesn't prove the Brane Theory. It is merely an observation which is *consistent* with what the Brane Theory predicts. All you would be able to say is that, within the limits of your observational error, the absence of WIMPS in your findings is more consistent with the Brane Theory compared to the Inflation Theory.

6. Knowing the Hubble Constant is important because it is one of the ways of determining distances to objects in the universe. This method is particularly important for determining distances to objects which are very far away, that is, at cosmologically relevant distances.

Distance determination is fundamental to cosmology, as well as astronomy in general. For instance, to understand the geometry of our universe requires accurate distance measurements. Also, since looking at objects farther away means looking at objects earlier in time, our understanding of the evolution (bottom-up vs. top-down) of large scale structures like clusters of galaxies depends on our distance measurements.

To determine H_0, we measure the recessional velocity v and distance d to many galaxies. We measure v using spectroscopic red-shifts (z), and we measure d using another distance indicator, such as Supernova Ia (SNe Ia) events which occurs in the galaxies. Plotting v on one axis and d on the other axis, we can calculate the slope of the line. This slope is the Hubble constant H_0.

$$v = H_o d \tag{5}$$

7. In order to find H_0, we need to use a method on a "lower rung" of the distance ladder, for instance, SNe Ia. As described in Question 6, one of the things we need to make a v vs d plot is to accurately measure the distances d to the galaxies we're studying. Finding those distances involves measuring the apparent brightness of SNe Ia events in those galaxies, which we can then use to find the object's distance using the flux relationship ($F = \frac{L}{4\pi d^2}$).

Furthermore, the SNe Ia method itself depends on the method of Cepheid variable stars, another method lower down on the distance ladder. To determine the absolute luminosities of the SNe Ia, astronomers had to find many examples at known distances. These SNe Ia events were in relatively nearby galaxies, such as in the Virgo cluster of galaxies. Using Cepheids, the distances to these galaxies can be found. Then, measuring the apparent brightness of the SNe Ia events, the actual luminosities of the SNe Ia could be found.

Furthermore, the method of Cepheids also depends on a method lower down on the distance ladder: parallax! That is, Cepheids had to be calibrated by observing them in a very nearby galaxy (the Large Magellanic Cloud), which distance could be determined by parallax.

So, the Hubble constant method lies at the top of a deck of cards, which we call the distance ladder. The Hubble constant method relies on the SNe Ia method, which relies on the Cepheid variable star method, which relies on parallax. An error at the bottom of the ladder accumulates and reaches the top. More on this in Question 10.

(See Appendix C for a listing of some of the methods used in the distance ladder.)

8.
Use the paper-line-finger method as presented by Prof. Ulmer in class.

(See Power Point slide for instructions.)

9. Human depth perception is about 50 to 60 feet.

10. The cosmic distance ladder consists of methods of determining distances in astronomy. At the lowest rung is the most trustworthy methods. Each successively higher rung is calibrated by the previous rung. The higher we go up the ladder, the less accurate the method becomes, because the errors at each rung accumulates. A significant correction of a distance measurement using a method lower on the distance ladder affects all measurements using methods of higher rungs.

We experienced this recently with HIPPARCOS (see Question 14). It refined parallax distances and caused a far-reaching, domino effect on all other distance measurements. It single-handedly increased our estimates of the age of the universe, as well as decreased the ages of the oldest objects in the universe.

(See Appendix C for a listing of some of the methods used in the distance ladder.)

11. Cepheids are not that bright. After a certain distance, they are too faint to reliably detect and measure. Also, Cepheids are rather rare stars. There are simply not that many of them around! We can reliable use Cepheids out to a distance of about 20 Mpc.

Interesting tidbit: Polaris, currently our "North Star", is a Cepheid variable. It is about 130 pc away.

Another interesting tidbit: Wanting to more accurately measure Cepheid variables in the Virgo cluster of galaxies motivated the creation of the Hubble Space Telescope. Using it, the Hubble Key Project team determined the best value of the Hubble constant to be 75 ± 8 km/s / Mpc.

12. By over-correcting for dust and looking at objects which he thought were dimmer than they actually were, Hubble *under-estimated* the distance to these objects, and hence *over-estimated* the Hubble constant.

13. A parsec = 3×10^{18}cm.

Also, a parsec = 3.26 light years.

14. HIPPARCOS = High Precision Parallax Collecting Satellite. It was designed to yield 100 times better star positions than obtainable from the ground. The improved accuracy of the star positions (out to about 200 pc) enabled astronomers to make much better distance determinations using parallax. In the process, it re-calibrated all the distance methods on higher rungs of the distance ladder and significantly corrected our view of the universe.

15. If an object were in a binary system, it would exhibit a characteristic "wobble" in its position. This wobble comes about because the star is orbiting around the common center of mass of the binary system. If the wobble is significant, the star will noticeably move in the sky. If the wobble is very small, it would show up in the spectrum of the star as a periodic red-shift/blue-shift. Since the Cepheids do not show such wobbling, they are not part of binary systems. Rather, they are individual stars which are unstable and thus pulsate periodically.

16. If dust were reducing the apparent brightness of an observed object, the light from the object would appear to be *reddened.* (Recall one of the essay questions on the 1st midterm exam.)

17. The Method of Cepheid Variable Stars:

i) Identify the object truly is a Cepheid by plotting its apparent brightness vs. time and recognizing the characteristic shape of the periodic pulsation.

ii) Measure the period of pulsation of the Cepheid from the above plot. The period of pulsation is simply the time it takes for the apparent brightness to go through one cycle, for instance, from peak brightness to the next peak brightness.

iii) With the pulsation period, find the Cepheid's intrinsic luminosity. We do this by simply using a plot of intrinsic luminosity-to-pulsation period. Such a plot can be created by studying a large number of Cepheids at a known distance.

iv) Having the intrinsic luminosity L and the apparent brightness (the flux F, which is simply how bright the object appears to us in the sky), we can calculate the distance d using the simple flux relationship $F = \frac{L}{4\pi d^2}$.

18. Making a v vs. d plot to determine H_0 requires that we are able to measure objects ...

i) bright enough to accurately determine v and d;

and

ii) numerous enough to get at least several dozen data points (the more the better).

It gets harder and harder to satisfy these requirements the further out in space we go. Practically speaking, the Hubble Relation is reliable out to $z \sim 0.1$, which is about 1 billion light years (1 Gly). It is the only method we have for measuring distances so great.

19. The Jackson-Faber Relation, the Tully-Fisher Relation, and the Fundamental Plane are all used to determine the intrinsic luminosity of a galaxy. The goal, of course, is to be able to use the intrinsic luminosity with the apparent brightness to find the distance to the galaxy using the flux relationship $F = \frac{L}{4\pi d^2}$.

The Tulley-Fisher Relation applies to spiral galaxies, using the width of the 21-cm line of neutral hydrogen as a measure of how fast a spiral galaxy is rotating. (See Question 22.)

The Jackson-Faber Relation applies to elliptical galaxies, using the velocity dispersion of stars close to the nucleus of an elliptical galaxy to measure how fast it is rotating. (See Question 23.)

The Fundamental Plane is a more sophisticated method based on the Jackson-Faber Relation. By plotting the luminosity, surface brightness, and spectral line width (which tells you the velocity dispersion) in a 3 dimensional plot, the measured values form a well-defined plane, and are not randomly distributed throughout the plot. Then, by measuring another galaxy's surface brightness and line width, we can refer to the Fundamental Plane to get its intrinsic luminosity (and hence its distance.)

20. Surface brightness does not depend on distance. So, the surface brightness will still be "A". (See below.)

21.

$$SB = \frac{F}{SA} \tag{6}$$

... where

SB is the surface brightness,

F is the flux $= \frac{L}{4\pi d^2}$, where d is the distance to the galaxy.

SA is the solid angle $= \frac{\text{Area of galaxy}}{d^2} = \frac{\pi r^2}{d^2}$, where r is the radius of the galaxy, and d is the distance to it.

Thus, for the problem, the relationship would be:

$$A = \frac{B}{C} \tag{7}$$

Notice that the distance to the object cancels out. That is,

$$\begin{aligned} SB &= \frac{F}{SA} && (8)\\ &= \frac{\frac{L}{4\pi d^2}}{\frac{\pi r^2}{d^2}} && (9)\\ &= \frac{L}{4\pi^2 r^2} && (10) \end{aligned}$$

So, the surface brightness is independent of the distance to the object.

(See Appendix D for an intuitive demonstration of why surface brightness is independent of distance.)

22. The Tully-Fisher (TF) Method uses the 21-cm line of cold, neutral hydrogen (H) in a spiral galaxy's disk. Normally, the line is very narrow. However, when we observe it from a spiral galaxy, the line is *broadened* due to the rotation of a spiral galaxy. That is, since the spiral galaxy is rotating about its center, one side of the galaxy appears to rotate *away* from us, while the opposite side appears to rotate *toward* us. By the Doppler effect, the 21-cm line from the H gas in the receding side will be red-shifted, while from the approaching side, it will appear blue-shifted. Combined, the total 21-cm will appear much *broader* than if it was emitted from a stationary source. The

amount of broadening tells us how fast the spiral galaxy is rotating. And how fast the spiral galaxy is rotating tells us how much mass (stars) is in the galaxy. Finally, how much mass (stars) is in the galaxy will tell us how intrinsically luminous it is. The intrinsic luminosity is what we want.

Here, again is the logic:

i) the broader the 21-line observed from a spiral galaxy, the faster it is rotating;

ii) the faster a spiral galaxy is rotating, the more mass (stars) it has;

iii) the more mass (stars) a spiral galaxy has, the greater is its intrinsic luminosity.

Why is the TF method only valid for spiral galaxies? Because only spiral galaxies have large amounts of cold, neutral H gas. Elliptical galaxies, another classification of galaxies, have very little if any H gas. Moreover, ellipticals have no spiral arms and have little internal structure of any kind, so there is no clear sense of rotation. (The Jackson-Faber Method, described in Question 23, is another method which can be used to gage an elliptical galaxy's intrinsic luminosity.)

Why does the TF method work best if the spiral galaxy is observed edge-on? The more edge-on we observe a spiral galaxy, the more obvious will be the contrast between receding and approaching sides of the galaxy. Imagine if you saw the galaxy face-on, not edge-on. Then, the galaxy will rotate in a plane perpendicular to your line of sight. That is, no part of the galaxy would appear to recede from you or approach you. Only when you see the galaxy edge-on will you be able to see one side receding and the other approaching. The more edge-on, the more the recession/approach will be obvious. This contrast between recession and approach of the spiral galaxy's arms is needed to broaden the 21-cm line, which then gives a good indication of the spiral galaxy's rotational velocity, and hence, its intrinsic luminosity.

23. What the Tulley-Fisher Relation is to spiral galaxies, the Jackson-Faber Relation is to *elliptical* galaxies. Instead of using the 21-cm line of H, the Jackson-Faber Relation uses the *velocity dispersion* of the stars close to the nucleus (the very central region) of an elliptical galaxy. The velocity dispersion is simply the average of the random velocities in the galaxy. (Remember, elliptical galaxies are not highly structured as spiral galaxies, so there's no obvious of "flow" of the stars around a central hub. They behave more like a

swarm of bees than a merry-go-round.)

The reasoning for the Jackson-Faber Relation is similar to that of the Tulley-Fisher Relation: the faster the velocity dispersion, the more mass is in the elliptical galaxy. The more mass is in the galaxy, the more intrinsically luminous it is. And once the intrinsic luminosity is found, the distance can be calculated from the flux equation $F = \frac{L}{4\pi d^2}$.

24. The theory for both the Tulley-Fisher and Jackson-Faber Relations is that the faster the motion of the stars or gas in the galaxy, the more mass (stars) it has, and hence, the more intrinsically luminous it is. See the last two questions for more discussion.

25. Radians are nice for small angles (less than 1 degree) because for small angles θ,

$$\theta \sim sin(\theta) \sim tan(\theta) \tag{11}$$

This only works if the θ is measured in radians, not degrees. (Try it on a calculator!)

Appendix A

Parallax Expressions

All parallax expressions are derived from simple ratios and geometry.

Let us derive the most general form, assuming the parallax angle is in units of *degrees*.

By equating ratios, we have:

$$\frac{p}{360^o} = \frac{R}{2\pi d} \tag{12}$$

... where

R = the length of your baseline, where the units are whatever is convenient (cm, km, AU, ly, etc.)

d = the distance to the object in the same units as R.

p = parallax angle measured in degrees.

There's nothing magical about this equation ... it simply says that the fraction of the little angle (parallax angle) to the entire circle (360^o) is the same as the fraction of the little length (baseline) to the total circumference of the circle ($2\ \pi$ d).

Solving for d, we get:

$$d = \frac{R\left(\frac{360^o}{2\pi}\right)}{p} \tag{13}$$

This is the equation to use if you're given a parallax angle in terms of degrees. And your units of distance d will be whatever your units of baseline R is.

If we want the equation for p in radians, we equate ratios:

$$\frac{p}{2\pi} \quad = \quad \frac{R}{2\pi d} \tag{14}$$

... where p is now in radians.

This simplifies nicely:

$$d = \frac{R}{p} \tag{15}$$

... and your answer for d will have whatever units that R has.

Finally, I derive the most simple expression as presented in the solution to Question 4. Since our parallax measurements are made from Earth, R is just 1 AU. So, let's find the distance to an object which subtends a parallax angle p of exactly *one arc second.*

There are 60 arc minutes in one degree, and 60 arc seconds in 1 arc minute. Thus, there are 3600 arc seconds in one degree. Or, one arc second is $\frac{1}{3600}$ of one degree. Using this for p in the Eq (12), we have:

$$\begin{aligned}
d &= \frac{R\left(\frac{360^o}{2\pi}\right)}{p} && (16)\\
&= \frac{1AU\left(\frac{360^o}{2\pi}\right)}{1 \text{ arc second}} && (17)\\
&= \frac{1AU\left(\frac{360^o}{2\pi}\right)}{\left(\frac{1}{3600}\right)^o} && (18)\\
&= \frac{1AU(360)(3600)}{2\pi} && (19)\\
&= 206,265AU && (20)\\
&= 1\text{parsec} && (21)
\end{aligned}$$

This is, in fact, the definition of parsec ... an object which subtends a parallax angle of 1 arc second (as viewed from a baseline of 1 AU) is 206,265 AU away. This distance is defined to be 1 parsec, and it is about 3.26 light years, or 3 x 10^{18}cm.

This leads us to the simple equation for parallax:

$$d = \frac{1}{p} \tag{22}$$

... where

d = the distance to the object in parsecs (pc). (1 pc = 3.26 ly)

p = parallax angle measured in arc seconds.

Notice, if the parallax angle is *larger*, the distance is *smaller*. Try it and see!

Appendix B

Comparing the Standard Big Bang/Inflation and Brane Theories

Big Bang/Inflation Theory	Brane Theory
Big Bang	"Bang"
Inflation	Radiation
Radiation	Matter
Matter	Dark Energy
Dark Energy ??	Contraction (Stagnation)
Future ???	"Crunch" / "Bounce"

Brief summary:

Big Bang/Inflation Theory:

The universe (space and time) had a definite beginning at the Big Bang. At this point, the temperature and density of the universe were infinite. Moments after the Big Bang, the universe exponentially expanded in size by a factor of 10^{+50} in only 10^{-32} seconds, powered by a cosmological constant. Quantum gravity was the dominant force during this inflation event, and so a spectrum of gravitational waves was created. After inflation, the universe resumed a more normal rate of expansion as it entered the radiation and matter eras. Currently, an unknown dark energy is causing the expansion to accelerate. The nature and origin of this dark energy are unknown, and hence, the ultimate future of the universe can not be predicted with certainty.

Brane Theory:

Preliminary information needed to appreciate the Brane story: *There are 10 dimensions in our universe, the 4 we can identify (3 spatial, 1 time dimensions), and 6 other dimensions. The extra 6 dimensions are separated into two domain walls, known as "branes" (for "membranes"), 3 dimensions on each brane. Our visible universe exists on one of these branes, called the "visible brane". The other brane interacts with the visible brane via gravity. However, we can't see the other brane, so the source of this gravitational interaction appears "dark" to us. Virtual exchanges of membranes between the boundaries of the branes cause a potential (related to a force), which is the source of the dark energy.*

The universe is infinite in both space and time. The universe goes through an endless cycle of expansion and collapse as the two branes repeatedly collide and bounce.

Let's follow one cycle.

First, the branes collide and bounce, producing the Big Bang, filling our universe with radiation and energy. The universe immediately enters the radiation and matter eras.

Then, the universe enters a long era of accelerated expansion fueled by dark energy and lasting trillions of years. As the universe expands, the distance between the two branes grow further apart, slowing the expansion.

At the end of the expansion, the universe is so large that it is essentially a vacuum. Contraction begins, although this phase will be observed in our visible brane as a *stagnation.* That is, the visible universe will not appear to shrink in spatial size. Rather, the contraction occurs in the extra dimensions not visible to us. The only effect in the visible universe would be changes in physical constants.

The contraction phase lasts trillions of years, allowing the universe to attain homogeneous, isotropic initial conditions. Density perturbation also have time to form, which become the seeds of large scale structure in the next cycle.

When the branes finally collide, the Big Crunch occurs, but the temperature and density are finite.

Then, the branes bounce, and the universe is again filled with radiation and energy, thus renewing the cycle.

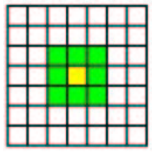
Appendix C

Distance Ladder

Method	Useful Within
Hubble's Relation	beyond 100 Mpc
Supernovae Ia	1 Gpc (= 1 billion ly)
Galaxy Relations (T-F, J-F, Fund. Plane)	200 Mpc
Variable Stars (Cepheids, RR Lyrae)	25 Mpc
Spectroscopic Parallax	10,000 pc
Parallax	200 pc
Radar	1 AU

(Ask me for a copy of this diagram, if you weren't in my class.)

The methods in the lower rungs of the ladder are more accurate. Errors increase as you go up the ladder. The methods in the upper rungs have the most uncertainties. Improvements in measurements to methods of lower rungs affect all distance determinations from methods in upper rungs.



Appendix D
Surface Brightness

To better understand why surface brightness is independent of distance:

Pretend we are looking at a square-shaped galaxy. (Of course, galaxies are not square shaped, but it helps to simplify their shape for this example.) When it is some distance d from us, it takes up a 3 x 3 area in our telescope, symbolized by the green region in the figure. That is, its apparent area is 9 square units. The apparent brightness (flux) of the galaxy is distributed over these 9 squares, so it would make sense to characterize the "brightness" of the galaxy as some average over the entire area. We call such a characterization the surface brightness. That is, for this simple example,

$$SB = \frac{\text{Flux at d}}{\text{9 squares}} \tag{23}$$

$$= \frac{F}{9} \tag{24}$$

Now, pretend we move that same galaxy 3 times as far away, that is, to 3d. Since flux is inversely proportional to the square of the distance, the flux will decrease to $\frac{F}{9}$. But at the same time, since the galaxy is now 3 times farther away, it also appears to shrink in size. In fact, at 3d, it will now appear to take up only 1 square box, colored yellow in the figure. That is, even though the flux diminished by a factor of 9, all the energy is now concentrated inside 1 box instead of 9. That is,

$$SB = \frac{\text{Flux at 3d}}{\text{1 square}} \tag{25}$$

$$= \frac{\left(\frac{F}{9}\right)}{1} \tag{26}$$

$$= \frac{F}{9} \tag{27}$$

So, at a farther distance, the decrease in flux is exactly counterbalanced by the decrease in apparent size, keeping the surface brightness constant. Thus, surface brightness is independent of distance.