David's Solutions to Homework Set 8, Cosmology, Fall 2002

1. (Same question as Problem 20, Homework Set 7.)

Surface brightness does not depend on distance. So, the surface brightness will still be "A". (See below.)

2. (Same question as Problem 21, Homework Set 7.)

$$SB = \frac{F}{SA} \tag{1}$$

... where

SB is the surface brightness,

F is the flux $= \frac{L}{4\pi d^2}$, where d is the distance to the galaxy.

SA is the solid angle $= \frac{\text{Area of galaxy}}{d^2} = \frac{\pi r^2}{d^2}$, where r is the radius of the galaxy, and d is the distance to it.

Thus, for the problem, the relationship would be:

$$A = \frac{B}{C} \tag{2}$$

Notice that the distance to the object cancels out. That is,

$$\begin{aligned} SB &= \frac{F}{SA} && (3) \\ &= \frac{\frac{L}{4\pi d^2}}{\frac{\pi r^2}{d^2}} && (4) \\ &= \frac{L}{4\pi^2 r^2} && (5) \end{aligned}$$

So, the surface brightness is independent of the distance to the object.

(See Appendix D of *Solution Set 7* for an intuitive demonstration of why surface brightness is independent of distance.)

3. The fundamental plane consists of 3 data points per galaxy: line width, luminosity, and *surface brightness.*

4. The data for galaxies had 3 rows per galaxy. One row was for the light (spectrum) from the galaxy. The other two rows were for the *calibration rows of standard, laboratory spectra.* The calibration spectra are made in a terrestrial laboratory, which is at relative rest. From these spectra, we can determine λ_e, that is, the wavelength which light from a particular atomic transition is *emitted* in a frame of reference at rest. Then from a galaxy's actual spectrum, we can measure λ_o, or the wavelength of light which is now *observed.* Thus, we can find the redshift:

$$z = \frac{\lambda_o - \lambda_e}{\lambda_e} \tag{6}$$

5. To measure a redshift using the sometimes "jumbled" galaxy spectra observed, we: (a) cross correlate it with a "standard spectrum" or a "template".

This correlation process acts like a filter. Pretend you are in a crowded room and everyone is talking at once. Obviously, it is very noisy, and you would find it difficult, if not impossible, to understand all the different conversations. But when you spot a couple of friends you know and make your way to them, you can easily focus in on their conversation and understand what they are saying. You are able to discern intelligible speech above the cacophony because you can tune out irrelevant information and focus in on the desired information which matches your interest. Likewise, given a rather messy spectrum (lots of people talking at once), we can correlate (compare) that to a standard spectrum (a couple of friends we know) to help us identify and focus on the relevant spectral features.

6. The concept needed to use the *brightest* globular clusters in a galaxy or the *brightest* galaxy in a cluster of galaxies as standard candles is that a *statistical maximum limit to brightness exists.* We simply assume that the brightest object has this maximum brightness (and thereby infer its absolute luminosity L). Then, we measure its apparent brightness F (flux), and using the flux relationship $F = \frac{L}{4\pi d^2}$, we can find the distance to the object.

The process is analogous to trying to find the distance to a group of people at some distance. Pretend we line up the whole class at some distance d away from you, and your goal is to find d. One way to figure it out would be to assume that the tallest person you see is 7 feet tall. Based on your experience with humans, you know very, very few people are taller than 7 feet! So, it's a very safe assumption that the tallest person you see won't be taller than 7 feet. He will probably be between 6 and 7 feet, but since this is just an estimate, we use 7 feet. Then, by measuring the angle at which he seems to appear to you, you can use simple trigonometry to figure out d. That is,

$$tan\theta = \frac{h}{d} \tag{7}$$

$$d = \frac{h}{tan\theta} \tag{8}$$

... where h is your assumed maximum height of the tallest person (7 feet), and θ is the angle which he appears to span from your position.

Of course, d will not be exactly right, because the tallest person is probably not exactly 7 feet tall. But at least you did not use 4 or 40 feet for h. So, you're not *too* far off. What you decided to use for h, while not exactly correct, is still a reasonable estimate, based on your statistical knowledge that there's a maximum limit to a human being's height.

Likewise, when we survey globular clusters in galaxies, or galaxies in clusters of galaxies, we notice that there's an upper limit of brightness for these objects. So, when we observe a new galaxy or cluster of galaxies and want to get an idea how far away it is, we assume that the brightest object we observe in the system is as bright as the maximum limit for brightness found in other systems. (This is analgous to assuming the tallest person in the class is as tall as the maximum height for humans.) Thus, the brightest object in these systems is assumed to be a *standard candle*, based on statistical maximums. Having the absolute luminosity (true brightness) and the flux (measured, apparent brightness), we then use the flux relationship to find the distance.

7. Using gravitational lensing to measure the Hubble constant relies on the true *distance* to the lensed object. Calculations involving the time-delay or angular distortion caused by a gravitational lens of known mass will give us the distance to the background (lensed) object. Then, by measuring the redshift, and hence velocity, of the lensed object, we can make a velocity vs. distance plot, the slope of which is Hubble's constant.

8. The Sunyaev-Zel'dovich (SZ) Effect involves the interaction of: (a) hot gas with the CMB.

What is the SZ effect?

As we learned before, clusters of galaxies contain a lot of hot gas at temperatures greater than 10^7 K. When photons from the cosmic microwave background (CMB) interact with the free electrons in this hot gas, they are scattered and gain energy. This process is called *inverse Compton scattering.* So, when we observe the CMB in the direction of such a cluster, we will observe a slightly distorted spectrum (plot of intensity vs. energy of the light) of the CMB compared to a direction which has no cluster of galaxies. This distortion is called the SZ effect, and it is caused by hot gas in a cluster of galaxies scattering CMB photons to higher energies.

What does the SZ effect teach us?

i) The SZ effect can be used to discover new clusters of galaxies. Significant distortions in the CMB consistent with the SZ effect reveals large conglomerations of hot gas.

ii) The SZ effect is also used to determine important physical characteristics of clusters of galaxies. The amount of distortion of the CMB spectrum depends on the *density*, *temperature* and *size* of the galaxy cluster. Knowing the *size* of the galaxy cluster and its *brightness*, we can determine its *distance*. And distance is the name of the game in cosmology. Distance is one ingredient in determining Hubble's constant (H), the other being recessional velocity as measured from the spectral redshift z.

iii) Using the SZ effect as an independent method of finding H has yielded values 35 to 85 km/(sec Mpc), consistent with values determined from other methods of finding H.

9. A good (rough-average-of-all-the-observational-evidence-to-date) value of the Hubble constant is: *65 km/(sec-Mpc).*

10. The Great Attractor is: (d) a 5 x $10^{16} M_\odot$ mass concentration in the direction of the Norma cluster of galaxies. (See Problem 11 for more discussion.)

11. By plotting the recessional velocities of relatively near-by galaxies and clusters of galaxies according to their distance from us (and thus making a "tadpole plot", where the length and direction of the tail of a tadpole represent the magnitude and direction of the recessional velocity of a certain galaxy), we discern a general flow toward a certain region in space, named the *Great Attractor* (GA), 65 Mpc away from us in the direction of the Norma cluster. All galaxies and clusters of galaxies (e.g. the Local Group, the Virgo Cluster,

the Hydra-Centaurus Supercluster) within 60 Mpc from us are all flowing toward the GA at about 600 km/s. It is as though we are all caught up in a current, forming a mass-migration of galaxies. From the dynamics of our flow toward the GA, we can calculate it must have a mass of about 5 x $10^{16} M_{\odot}$, which is much, much greater than the luminous, normal matter we observe to exist there. Thus, the mass at the GA must be *dark*, or non-luminous *dark matter*.

12. The rotation curve is a plot of stars' rotational velocities vs. distances from the center of a galaxy. A star's rotational velocity is simply how fast it is traveling around the center of the galaxy.

If the only matter in the galaxy is normal visible matter and this matter behaves according to simple Newtonian gravity ($F_g = \frac{GMm}{r^2}$), we expect that up to a certain distance from the center of the galaxy, a star's rotational velocity increases. This is the behavior of the *uniform disk model*. Beyond the visible "edge" of a galaxy, very few stars can be seen, and so very little luminous mass exists. Thus, the rotational velocity of any object (or gas) beyond this point should begin to decrease. Further and further away, the rotational velocities should get smaller and smaller. This is the *Keplarian model*.

However, when we create rotation curves from observations of stars in many galaxies, a cosmic conspiracy is revealed! The expected Keplarian decrease in velocity does not exist! Rather, the velocities flatten out and remain constant out to great distances, far beyond the outer visible "edge" of a galaxy. Somehow, mass is distributed in such a way as to keep rotational velocities constant at these distances. It would seem as if nature has conspired to distribute mass in this rather peculiar way. Furthermore, because we don't observe this matter, it must be *dark*. In fact, this discovery (by astronomer Vera Rubin) of flat galactic rotation curves was the first evidence that *dark matter* might exist.

13. We use *21-cm emission lines from cold hydrogen gas* out beyond the visible "edge" of a galaxy to make rotation curve measurements. (This is the same method used in the Tully-Fisher Method; review Problem 22, Homework Set 7.) The flat rotation curve tells us: (d) the galaxy has a great deal of mass in the form of dark matter than extends beyond the galaxy's visible edge.

14. Based on the uniform disk model and the Keplarian model, the rotational velocity as a function of distance should: (a) go up and go down.