

David's Solutions to Homework Set 9, Cosmology, Fall 2002

1. The study of shapes is also called *morphology*.
2. The main types of galaxies: *spiral, barred spirals, S0, elliptical, irregular*.
3. Hubble tuning fork diagram: see Ulmer's Power Point slide "Edwin Hubble's Classification Scheme". The ellipticals are on the "handle" of the tuning fork, and the spirals are on the two "prongs". The S0 type of galaxy is at the "vertex" between the handle and the prongs.

Hubble thought ellipticals were formed first and then evolved into spirals.

4. The S0 type of galaxy has no current star formation, relatively little gas and dust, but does have a disk.

5. The Hubble Tuning Fork Model postulates that ellipticals evolve into spirals. However, we now believe this is very unlikely. Two main lines of evidence lead to this hypothesis:

i) Rotation (Angular Momentum)

Ellipticals are not rotating very much. That is, their *angular momentum* is very small. On the other hand, spirals *are* significantly rotating and have a substantial angular momentum. If ellipticals truly evolved into spirals, they would require some external torque (force) to spin them up. No known mechanism would naturally accomplish this. Thus, ellipticals probably do not evolve into spirals.

ii) Gas and Dust

Ellipticals have very little gas and dust. We think that all the free gas and dust were used up to form stars in the past. Very little star formation currently occurs in ellipticals. However, spirals have plenty of gas and dust. A lot of star formation occurs in spirals. Thus, ellipticals would need to somehow repopulate itself with gas and dust to trigger star formation if they truly evolved into spirals. Again, no known mechanism naturally accomplishes this. Thus, ellipticals probably do not evolve into spirals.

6. Be able to recognize the Andromeda Galaxy (M31), the Large Magel-

lanic Cloud (LMC), and the Small Magellanic Cloud (SMC). (The pictures will be projected onto the screen during the final exam, and you will have to identify them.)

The LMC and SMC are irregular galaxies about 50,000 pc away which are gravitationally bound to our galaxy, the Milky Way. In fact, they are so close that the gravitational forces from the Milky Way are tearing them apart. They are only observable from the southern hemisphere.

The Andromeda Galaxy is the closest spiral galaxy to the Milky Way. It is actually approaching us and some astronomers believe it will collide into the Milky Way many millions of years in the future. You can actually see the Andromeda Galaxy with your naked eye, even from Northwestern (which suffers from poor, light-polluted skies.) It is one of the objects we have been observing with the Dearborn telescope this past fall, and will be visible all winter. Through the telescope, we can see its bright core. Amazingly, the Andromeda galaxy is so large in size that it is 6 times wider than the image of the full moon! But of course, it is much fainter, so most people don't ever notice it.

7. M87, 3C273, and "West Side Story" all have *jets*.

(See Appendix J for the lyrics to *The Jets*. :) )

8. QSOs are thought to be powered by massive black *holes*.

9. QSOs may be some of the most luminous objects in the universe. They are very bright in the radio frequencies but appear as point-sources of light in the optical frequencies. Thus, they are also called *quasars*, which stands for quasi-stellar-*radio*-sources. They have very high redshifts, suggesting they are receding very rapidly away from us (at significant fractions of the speed of light!). By the Hubble relation, the high redshifts suggest they are very far away. In order for us to even see them, their intrinsic luminosity must be extremely great. Given their apparent brightness (flux) and their distance (as determined from their redshift), we estimate that QSOs are about 1 *trillion* times as bright as the Sun! This is about 10-100 times more luminous than most galaxies. So, QSOs produce a *lot* of energy!

At the same time, the flux of QSOs varies significantly in a period of about a week. By the principle of causal contact (see Appendix B to help you understand this better), the size of the object must thus be smaller than about

a light-*week* (the distance light travels in a week). In contrast, a typical galaxy is about 100,000 light-*years* in diameter! So, QSOs are extremely compact objects.

We cannot account for the huge energy output of QSOs with normal nuclear burning. The only way to release so much energy from such a small object is by matter falling onto a supermassive black hole. Supermassive black holes are celestial monsters having masses more than  $10^6 M_{\odot}$ , which accrete surrounding gas, dust, and stars onto it. They reside at the center of nearly all spiral and elliptical galaxies, including our own Milky Way.

10. QSOs stand for *Quasi-Stellar Objects*, that is, (d) they look like stars in celestial photographs. Another commonly used term for QSOs is *quasars*.

11. QSOs are as luminous as 10-100 galaxies put together (about one *trillion* suns!), but are (b) 100-1000 times smaller than the distance to the closest star to the sun.

The period of the variation in the luminosity of QSOs is on the order of a week, suggesting the sizes of QSOs are also on the order of a light-week (see Problem 9). The distance to the closest star (Proxima Centauri) to the sun is about 4 light-*years*. Let's just say there are 50 weeks in a year. Thus, there are about 200 light-weeks in 4 light-years. So, if a QSO is one light-week in size, then it is 200 times smaller than the distance of Proxima Centauri to the Sun.

12. *Jets* somewhat resemble nuclear bomb blasts.

13. The spin axis of a massive black hole in QSOs and radio galaxies defines the direction of the *jets*.

14. Galaxy formation in a nutshell:

(Note: How galaxies form is highly speculative at this stage in astronomy. We are far from having the complete picture. Nearly every astronomy text offers a slightly different sequence of galaxy formation! The following synopsis summarizes one possible scenario, the one which Prof. Ulmer presented.)

Since nearly all spiral and elliptical galaxies today have supermassive black

holes at their cores, we believe that all such galaxies began with massive black holes at their centers. These black holes may have been the remnants of extremely massive stars which constituted relatively small, irregular galaxies. These irregular galaxies merged together to form larger galaxies, and the massive black holes all sank toward their common center to form even more massive black holes. Over time, the black holes grew in size as they accreted surrounding matter, eventually becoming supermassive black holes with millions of solar masses.

Here, galactic evolution branches off into two possibilities, primarily based on the galaxy's rotation (angular momentum).

i) High rotation (angular momentum) ... Spiral Galaxies:

Galaxies rotating faster resist the accumulation of more matter, since the centripetal force due to the rotation tends to push matter away. These galaxies develop flattened disk shapes due to the high rotation (like pizza dough gets flattened when the chef spins it!). Since less mass is available as fuel for the black holes in the nuclei, these galaxies produce less energy and thus evolve into weaker active galactic nuclei (AGN) and weaker QSOs. (Remember, the infall of matter onto the central black hole fuels the quasar.) Eventually, these galaxies evolve into spiral galaxies.

ii) Low rotation (angular momentum) ... Elliptical Galaxies:

Galaxies with slower rotation rates have much less outward centripetal force, thus allowing them to accrete much more matter in a shorter period of time. This provides the central black hole with a lot of fuel. Thus, these become the brightest QSOs, the most active AGNs, and show the largest jets. Eventually, these galaxies evolve into elliptical galaxies. Since they are not rotating very fast, they have a more rounded, filled-out shape, in contrast to the flattened, disk shape of spirals.

In both cases, as the fuel is used up in time, the QSOs begin to die out, that is, become weaker and fewer in number. Thus, the smaller the redshift, the fewer the QSOs we observe, until in the present era, there are no more QSOs.

Finally, S0 galaxies may result from the merger of spiral galaxies. The disk shape is preserved, but the galactic collision propelled all the intergalactic gas and dust away, leaving very little in the final merged galaxy. Interestingly, both the Milky Way and Andromeda Galaxies are spiral galaxies. Some astronomers predict a collision between the two in the future. Will our galaxy eventually

merge with Andromeda to become an S0 type galaxy?

15. Halton Arp and the Burbidges are the "nay-sayers" who argue that (b) redshifts don't tell us the distance to the QSOs.

The individual QSOs in some groups of QSOs seem to be visually associated, but each are observed to have vastly different redshifts. So, if the apparent association is not just an optical illusion, they must all be at roughly the same distance from us. If they are, then the fact that they have very different redshifts suggests that redshifts do *not* tell us their distance via the Hubble Relation.

Furthermore, QSOs spectra reveal large amounts of heavy metals such as iron, in fact, up to three times greater than what we see from a typical nearby galaxy. In the current framework of stellar evolution, a lot of time is necessary to produce heavy metals like iron. Taken at face value, the redshifts of some QSOs indicate they existed when the universe was only 1 billion years old, and it is difficult to imagine how they could form so much iron in that amount of time. Thus, either in the early universe some yet unknown mechanism produced heavy metals in large amounts very quickly, or QSOs are really not that far away (and hence exist in an older universe). That is, the distance to the QSO based on its redshift may be incorrect. Something having nothing to do with the expansion of the universe may be causing the large redshifts in QSOs. Thus, a QSO may have a large redshift even when it is not that far away in space or time.

... at least, this is what Halton Arp and the Burbidges would have us believe!

16. *Molecular and ionized forms of H* will be missing from a study of the 21 cm line in the interstellar medium. Recall that the 21 cm line in H is formed when a spin-flip occurs between an electron and proton in the neutral H atom. Ionized H has lost its electron, so ionized H cannot form 21 cm radiation. Likewise, molecular H introduces one more electron and one more proton, complicating the electron orbitals sufficiently to mask 21 cm line emission.

17. The mass-to-light ratio (M/L) for our own galaxy does not need to be scaled with "h" (where  $h = \frac{H_0}{100}$ ) because (b) we measure distances within our galaxy using parallax or Cepheids, not redshift. (See Appendix A for the mathematical details relating M, R, and L to h.)

18. High M/L values compared to local values argues for the existence of *dark matter*. A high M/L value simply means that an object or region is much more massive than it is luminous. So, if an object is "dark", its M/L value should be very large.

Some typical M/L values:

Region or Object	M/L (units of $M_{\odot}/L_{\odot}$ )
Local Neighborhood (100 pc from Sun)	5
Milky Way Galaxy	20
External spiral galaxies	40
The Great Attractor	500-1000

19. Given a fixed set of measurements, arbitrarily increasing the Hubble constant by a factor of two does *not* change  $\Omega_m$ , so (d) it remains the same. Density is proportional to  $H_0^2$ . But  $\Omega_m$  is a ratio of densities, that is  $\Omega_m = \frac{\rho}{\rho_c}$ . Since density is in both the numerator and denominator, the Hubble constants cancel out for the whole ratio. So,  $\Omega_m$  is independent of the Hubble constant.

(See Appendix A for the mathematical details showing that density is proportional to  $H_0^2$ , and thus that  $\Omega_m$  is independent of  $H_0$ .)

20. The lower plot represents a universe without a cosmological constant, that is,  $\Omega_{\Lambda} = 0$ . Thus, in this universe, there is only mass, and only  $\Omega_m$  is relevant. The upper plot represents a universe *with* a cosmological constant, such that  $\Omega_t = \Omega_m + \Omega_{\Lambda} = 1$ . Thus, for this plot, as  $\Omega_m$  gets smaller,  $\Omega_{\Lambda}$  grows larger to keep  $\Omega_t$  equal to 1. These two plots must cross at  $\Omega_m = 1$ , simply by definition. That is, for the upper plot, when  $\Omega_m = 1$ , then  $\Omega_{\Lambda} = 0$ , since for that plot,  $\Omega_t = 1$ . But, for the bottom plot,  $\Omega_{\Lambda} = 0$  *always*. So, at  $\Omega_m = 1$ , the two plots *both* represent the case when  $\Omega_{\Lambda} = 0$ , and so must share the same point, namely, the point where the plots intersect.

The figure shows that (c) lower values of  $\Omega_m$  correspond to greater values of  $t/H_0^{-1}$ . This is simply saying that for a given value of the Hubble constant  $H_0$ , the age of the universe increases when  $\Omega_m$  decreases. (Recall Problem Set 5, Question 10.)

21. The double lobed radio source diameter makes a poor standard ruler for distance measurements because (b) the size of the rulers evolves with time (and we don't understand the evolution of these systems very well).

22. Given an ore sample that is  $1/8$  uranium and  $7/8$  lead, the sample would be 3 half-lives old.

After 1 half-life, a sample will have  $1/2$  uranium. After 2 half-lives, it will have  $1/2$  of what it started out with after 1 half life, that is  $1/2 * 1/2 = 1/4$  uranium. And after 3 half-lives,  $1/8$ . After 4,  $1/16$ . After 5,  $1/32$ . Etcetera, etcetera.

## Appendix A

### Why Does $\rho \sim H_0^2$ ?

To demonstrate why density is proportional to  $H_0^2$ , recall that  $\rho = \frac{M}{V}$ , where M is the mass and V is the volume. Take the volume to be that of a sphere, that is  $V = \frac{4}{3}\pi R^3$ . So, if we can determine how M and R relate to  $H_0$ , then we will be able to find how  $\rho$  depends on  $H_0$ .

First, let us relate R to  $H_0$ . R is simply the radius of the observable universe. It is a distance. We can relate distance R to the Hubble Constant via the Hubble Relation:

$$v = H_0 D \quad (1)$$

$$v = H_0 R \quad (2)$$

or,

$$R = \frac{v}{H_0} \quad (3)$$

So, we see that the R is inversely proportional to  $H_0$ , or

$$R \sim \frac{1}{H_0} \quad (4)$$

Next, we proceed to find the relationship between M and  $H_0$ . Equate the centripetal force with the gravitational force:

$$F_c = F_g \quad (5)$$

$$\frac{mv^2}{R} = \frac{GMm}{R^2} \quad (6)$$

or, after solving for M:

$$M = \frac{v^2 R}{G} \quad (7)$$

So, M is directly proportional to R, and thus by what we found from Eqn (4), inversely proportional to  $H_0$ . That is,

$$M \sim R \quad (8)$$

$$\sim \frac{1}{H_0} \quad (9)$$

In summary, both  $R \sim \frac{1}{H_0}$  and  $M \sim \frac{1}{H_0}$ . Now, we can find how  $\rho$  is related to  $H_0$ .

$$\rho = \frac{M}{V} \quad (10)$$

$$\sim \frac{M}{R^3} \quad (11)$$

$$\sim \frac{\frac{1}{H_0}}{\frac{1}{H_0^3}} \quad (12)$$

$$\sim H_0^2 \quad (13)$$

Thus,  $\rho \sim H_0^2$ .

Finally, to show that  $\Omega_m$  is *independent* of  $H_0$ :

$$\Omega_m = \frac{\rho_0}{\rho_c} \quad (14)$$

$$\sim \frac{H_0^2}{H_0^2} \quad (15)$$

$$\sim \text{independent of } H_0 \quad (16)$$

For the dependence of  $M/L$  on  $h$  ( $= \frac{H_0}{100}$ ), we recall that  $M$  and  $R$  are proportional to  $\frac{1}{h}$ . Moreover, the flux relationship shows that  $L = F4\pi d^2$ , where  $d$  is the distance and thus analogous to  $R$ . Thus,  $L$  is proportional to  $R^2$ , that is,

$$L \sim R^2 \quad (17)$$

$$\sim \frac{1}{h^2} \quad (18)$$

$$(19)$$

Thus

$$M/L \sim \frac{\left(\frac{1}{h}\right)}{\left(\frac{1}{h^2}\right)} \quad (20)$$

$$\sim h \quad (21)$$

## Appendix B

### Why Fluctuation Periods Set Limits on Size

An often used principle in astronomy is that an object cannot flicker in a period of time less than light takes to cross it. This principle is based on the finite speed of light and thus the idea that for any two points to be causally related, they must be close enough together for light to travel between them in the desired amount of time.

First, let us define what we mean by "flickering" of light. It is a fluctuation in the intensity of light, for instance, when a source of light grows noticeably brighter and then darker and then brighter and then darker, etc. If the time interval between the cycles of brightness and darkness is small, the flickering appears rapid, and the object might be described as "twinkling" (Twinkle, twinkle little star ...). If the variations are longer, we may not notice them unless we carefully keep track of the brightness of the object over a period of time. For instance, the Cepheid variable stars fluctuate over longer time intervals, and we would not notice their variation unless we observed them over several hours or days.

QSOs produce a tremendous amount of light, and interestingly, their brightness varies by as much as a factor of 2 over a period of about 1 week. That is, if we plotted a QSO's apparent brightness versus time over a period of several weeks, we would find it gets brighter and dimmer with a period of about a week. It is "flickering" over a period of a week. Moreover, the brightness variations are sharply defined and not blurred. We could definitely notice when it was brighter compared to when it was dimmer.

Here is the proposal: *An object must be smaller than the distance traveled by light in the period of its flickering.*

That is, if an object flickers on the order of 1 second, it must be smaller than 1 light-second in size. Or, in the case of QSOs, since the period of flickering is about 1 week, the size of QSOs must be at the most 1 light-week.

What does flickering have anything to do with the size of an object?

An analogy may help. Instead of *light*, let us think about *sound*, which also has a finite speed. Since the speed of sound is much less than the speed of light, we are more familiar with effects due to the delay in sound arrival times, for instance, as echoes ... echoes ... echoes ...

Imagine you are with a band of 100 trumpet players. You are positioned such that the trumpet players are lined up directly away from you, that is, you can really only see the closest trumpet player, and the others are behind her in relation to you. So, basically, you and the trumpet players are all in the same single-file line. Let's number the closest trumpeter T1, and the farthest one, T100. So, the line would look something like this:

You . T1 T2 T3 ... (all the other trumpeters) ... T98 T99 T100

Now, let the line of trumpeters be 300 meters (about the length of a 3 football fields). Since the speed of sound is about 300 m/s, it takes sound 1 second to get from T1 to T100.

So, if the band blasted their trumpets all at once for 10 seconds and stopped, you would actually hear *11* seconds of trumpet blasts, because it took an extra second for the sound from T100 to reach you.

Now, let the band play all together in a staccato fashion, that is, let each play a single note but pausing and re-playing very rapidly (in a fraction of a second) to make a choppy effect. But let them play this staccato song all together in unison.

Would you be able to hear the staccato effect? That is, could you clearly distinguish the rapid variations in the intensity of sound? No. Why? Because the sound from a trumpet blast farther away will reach you when a pause from a closer trumpet is occurring, thus masking over the staccato effect of the ensemble. You would hear a continuous sound of trumpets blaring. You would not be able to detect any "flicker" in the sound.

In fact, you won't be able to clearly distinguish any significant fluctuation in the trumpet blasts until the time between the pauses was around 1 second long, the "sound-length" of the line of trumpeters. If the pause is 1 second long, you will be able to experience at least a moment in which the entire line seems to have stopped playing. And immediately afterward, the cycle will start over again. So, in order for you to experience a flickering effect in the sound, the flickering period must be at least as long as the source of the sound, in this example, the length of the line. Conversely, if you could detect a flickering in the sound, you would know that the length of the line must be smaller than the distance in which sound travels during the period of the flickering.

Let us apply this analogy to an astronomical object. In terms of the analogy, the trumpeters are the atoms on the astronomical object, the length of

the line is the size of the object, and instead of trumpeters producing sound from trumpets, the atoms are emitting radiation (light) from atomic processes. So, if you can detect a flickering in the light from an astronomical object, you know it must have a size which is smaller than the distance which light can travel in the period of the flickering.

In the case of light, the situation is more restrictive than sound, because nothing can travel faster than the speed of light. That is, if something is going to flicker, it has to grow brighter and dimmer in a definite, periodic fashion. But one piece of the object must somehow "know" when to get brighter and dimmer with another piece on the opposite side of the object. That is, one piece of the object must be in *causal contact* with other pieces of the object. The fastest way to convey that information is at the speed of light. Practically speaking, the actual speed of the communication would occur at far lesser speeds, but we'll give it the benefit of the doubt, use the maximum possible speed, and thus estimate its *maximum* possible size. So, if an object flickers, the size must be smaller than the distance in which light travels in the fluctuation period.

To recap, the fluctuation period of an object tells us the maximum size the object can be. This is because the speed of light is finite, and so it takes time for light signals to travel to you from a more distant part of the object compared to a closer part. Moreover, since the speed of light is the fastest way to communicate information, the time it takes for light signals to convey coherent periodicity which characterizes flickering tells us the maximum size the flickering object must be. Thus, in the case of QSOs, flickering periods on the order of a week tells us they must be less than a light-week in size. This is a very small region when we consider their energy output is equivalent to 10-100 average galaxies, which are hundreds of thousands of light-*years* in size.

Appendix J

*The Jets*

from the musical, *West Side Story*

music by Leonard Bernstein and words by Stephen Sondheim

When you're a Jet,  
You're a Jet all the way  
From your first cigarette  
To your last dyin' day.

When you're a Jet,  
If the spit hits the fan,  
You got brothers around,  
You're a family man!

You're never alone,  
You're never disconnected!  
You're home with your own:  
When company's expected,  
You're well protected!

Then you are set  
With a capital J,  
Which you'll never forget  
Till they cart you away.  
When you're a Jet,  
You stay a Jet!

Oh, when the Jets fall in at the cornball dance,  
We'll be the sweetest dressin' gang in pants!  
And when the chicks dig us in our Jet black ties,  
They're gonna flip, gonna flop, gonna drop like flies!

When you're a Jet,  
You're the top cat in town,  
You're the gold medal kid  
With the heavyweight crown!

When you're a Jet,  
You're the swingin'est thing:  
Little boy, you're a man;  
Little man, you're a king!

The Jets are in gear,  
Our cylinders are clickin'!  
The Sharks'll steer clear  
'Cause ev'ry Puerto Rican's a lousy chicken!

Here come the Jets  
Like a bat out of hell.  
Someone gets in our way,  
Someone don't feel so well!

Here come the Jets:  
Little world, step aside!  
Better go underground,  
Better run, better hide!

We're drawin' the line,  
So keep your noses hidden!  
We're hangin' a sign,  
Says "Visitors forbidden"  
And we ain't kiddin'!

Here come the Jets,  
Yeah! And we're gonna beat  
Ev'ry last buggin' gang  
On the whole buggin' street!  
On the whole!  
Ever!  
Mother!  
Lovin'!  
Street!

Yeah!